

## Transition Zone Modeling

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This lecture reviews current practice as well as new modeling ideas for the calculation of at least skin friction and heat transfer between the onset and end of transition.

The principle means of transition zone modeling dating from the 1950's is through the intermittency models developed by Narasimha and colleagues. In these methods the length of the transition zone is closely correlated to mean boundary layer properties at the onset location and so the procedure is critically dependent on a correct determination of the onset location. The procedure is independent of the level or character of the free-stream disturbances implying independence of the spot generation mechanism or other detailed physics. Determination of onset location is principally by correlation methods of by  $e^N$  for quiescent streams. Recent development of PSE methods by Herbert and others provides a powerful method for calculating onset at all turbulence levels. Additional data for modeling of bypass transition over heated flat plates have been obtained by Sohu and Reshotko in the NASA-Lewis boundary layer channel. With a series of grids placed upstream of the contraction, free stream turbulence levels ranging from 0.4% to 6% can be produced. For  $Tu=0.4\%$ , heating the plate advances transition as would be expected from T-S considerations. For  $Tu>1\%$ , there is negligible effect of heating on transition.

Rai and Moin (1990) have performed a direct Navier-Stokes (DNS) simulation of the grid 2 case of Sohu and Reshotko. This calculation which used about 8000 hrs. of CRAY-YMP time reproduced the onset of transition and most profile features through transition but missed the turbulent skin friction level. Such DNS simulation have to do better before they can be used as validation "experiment" for simpler modeling methods.

Sohu's intermittency conditioned mean profiles show that the nonturbulent part is Blasius near the beginning of transition but is fuller than Blasius as transition progresses. The turbulent part is initially less full than the standard turbulent boundary layer but approaches the Musker profile as transition is completed. Sohu's boundary layer spectra for Grids 2 and above are unambiguously turbulent. The spectra for Grid 0.5 and Grid 1 are somewhat ambiguous but our sense is that the bypass mode is dominant. Sohu's spectra have some t-s features. Suder's spectra in the same facility for Grid 0 without plate heating fairly clearly document a T-S

path to transition. The turbulent spectra observed in bypass situations suggest that bypass transition might be modeled using turbulent flow methods. Additional experiments are under way for boundary layers with pressure gradients.

The balance of the presentation shows the experience to date of the Wu and Reshotko with a multiple-time-scale  $k$ - $\epsilon$  method. The skin friction results for the ERCOFTAC T3A case ( $Tu=2.8\%$ ) are reasonable but subject to improvement. The distributions of the  $k$  and  $\epsilon$  components through transition are shown. More specifically, the low frequency component of  $k$  ( $k_p$ ) through transition is comparable to what is expected from such experiments including law-of-the wall similarity in the near-wall region. For turbulence levels above 12%, the law-of-the-wall similarity breaks down indicating that the outside turbulence is strong enough to change the near wall behavior thus altering the characters of the turbulent boundary layer at sub-elevated turbulence levels.

Work is continuing toward improving the MTS modeling and making the procedure more robust. Also it is important that the receptivity issue be examined toward identifying the physics of bypass initiation. Of particular interest in this regard are the forced algebraically growing subcritical "modes" identified recently by Henningson, Trefethen and colleagues.

# INTERMITTENCY MODELS

$\gamma = \gamma(x)$  { Narasimha  
Arnal  
Chen and Thyson

Linear combination  
 $( ) = (1-\gamma)( )_L + \gamma( )_T$

Algebraic  
 $\gamma = \gamma_m + \gamma(x) \gamma_T$

Narasimha

$$\gamma = 1 - \exp \left[ - \frac{(x - x_e)^2 n \sigma}{U} \right]$$

$x_e$  is onset location

$n$  is spot formation rate

$\sigma$  is a spot propagation parameter

$$\frac{n \sigma}{U} = \frac{0.41}{\lambda^2} = \frac{N}{\theta_e^2} \left( \frac{\gamma}{U \theta_e} \right) \left\{ \lambda = \sqrt{\frac{0.41}{N}} \theta_e R_{\theta_e}^4 \right.$$

where  $\lambda$  is distance between  $\gamma = 0.75$  and  $\gamma = 0$ .

$N$  is dimensionless parameter thought to be nearly constant

$\theta_e$  is momentum thickness at  $x_e$

# DETERMINATION OF ONSET

|   |   |
|---|---|
| Quiescent Streams<br>$0 < Tu \leq 0.4\%$              | Correlation, PSE, $e^N$ , *               |
| Weak Bypass ("Dangerous")<br>$0.4\% \leq Tu \leq 2\%$ | Correlation, PSE, *                       |
| Strong Bypass<br>$Tu \geq 2\%$                        | Correlation, PSE, * $k-\epsilon$<br>(DNS) |

\* Methods based on Henningson, Trefethen inspired techniques involving non-orthogonal forced modes, principally 3D!!

# WHAT ARE THE ALTERNATIVES ?

Quiescent Flows - Wilcox:  $e^4$  followed  
by  $g-\tau$

Bypass Flows - Many investigators:

Turbulent Flow  
methods

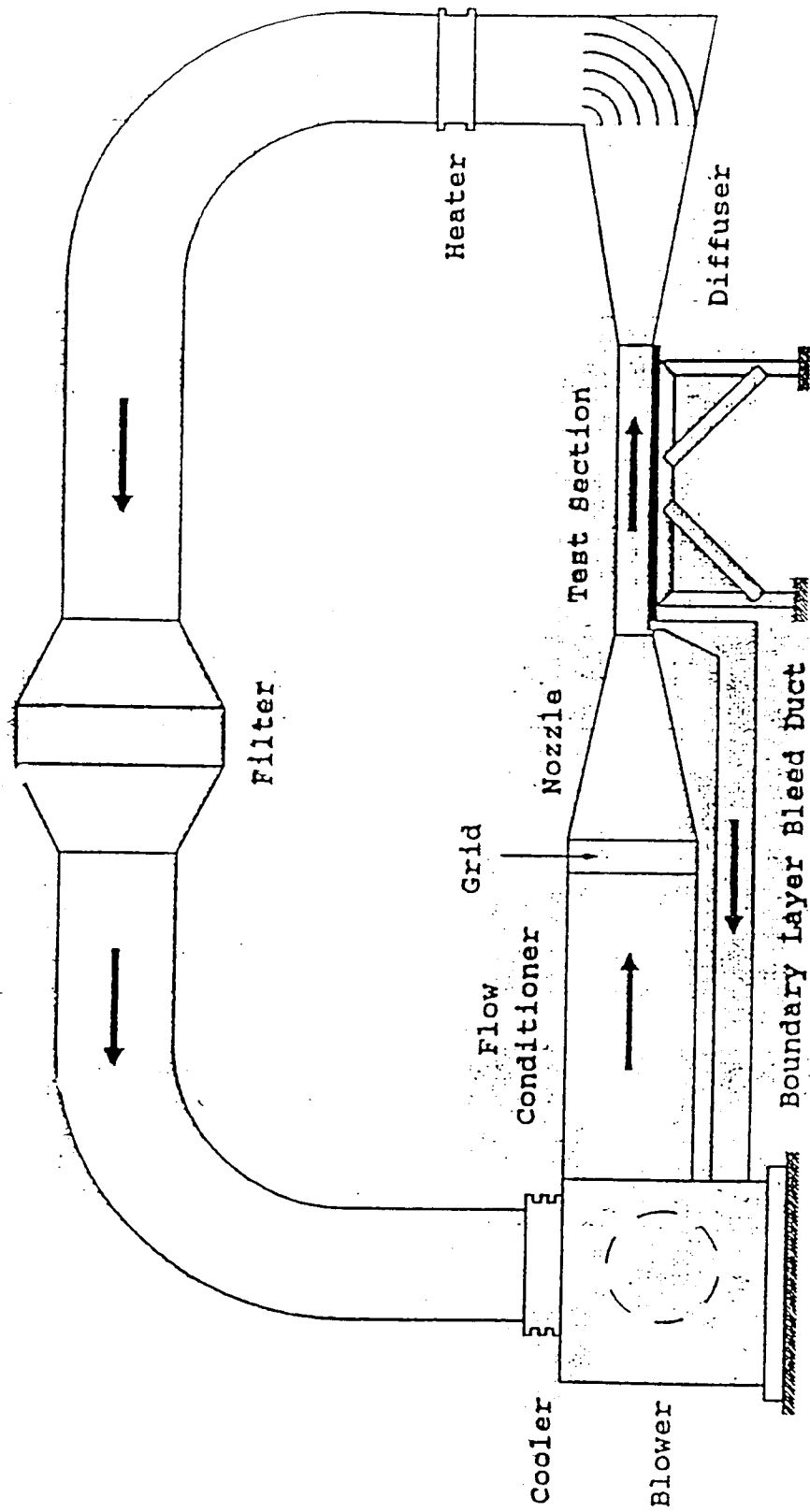


Fig. 1 Schematic diagram of the wind tunnel

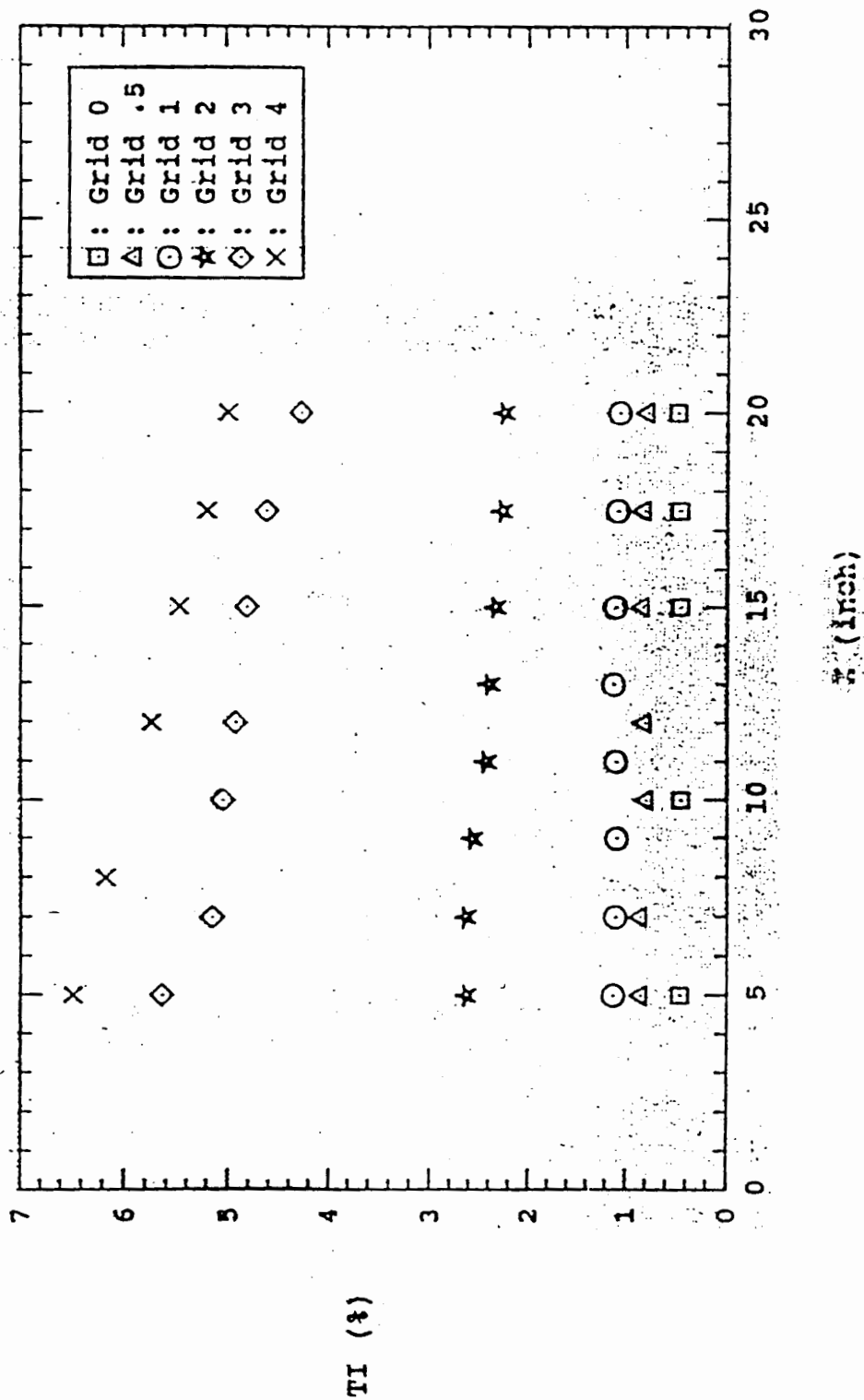
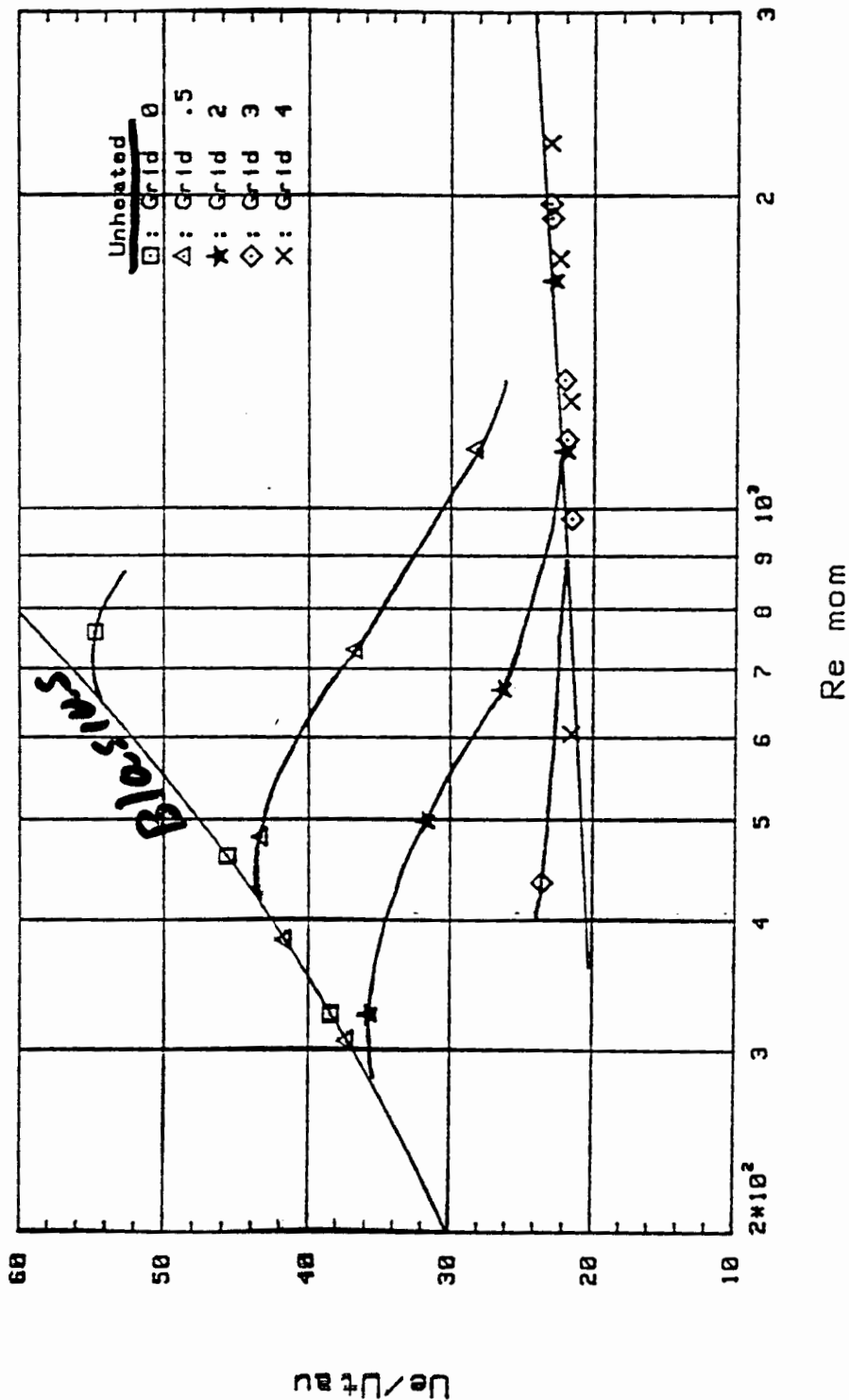


Fig. 19 Streamwise freestream turbulence intensity through the test section

SOHN

$$\frac{u_e}{u_r} = \sqrt{\frac{2}{5}}$$

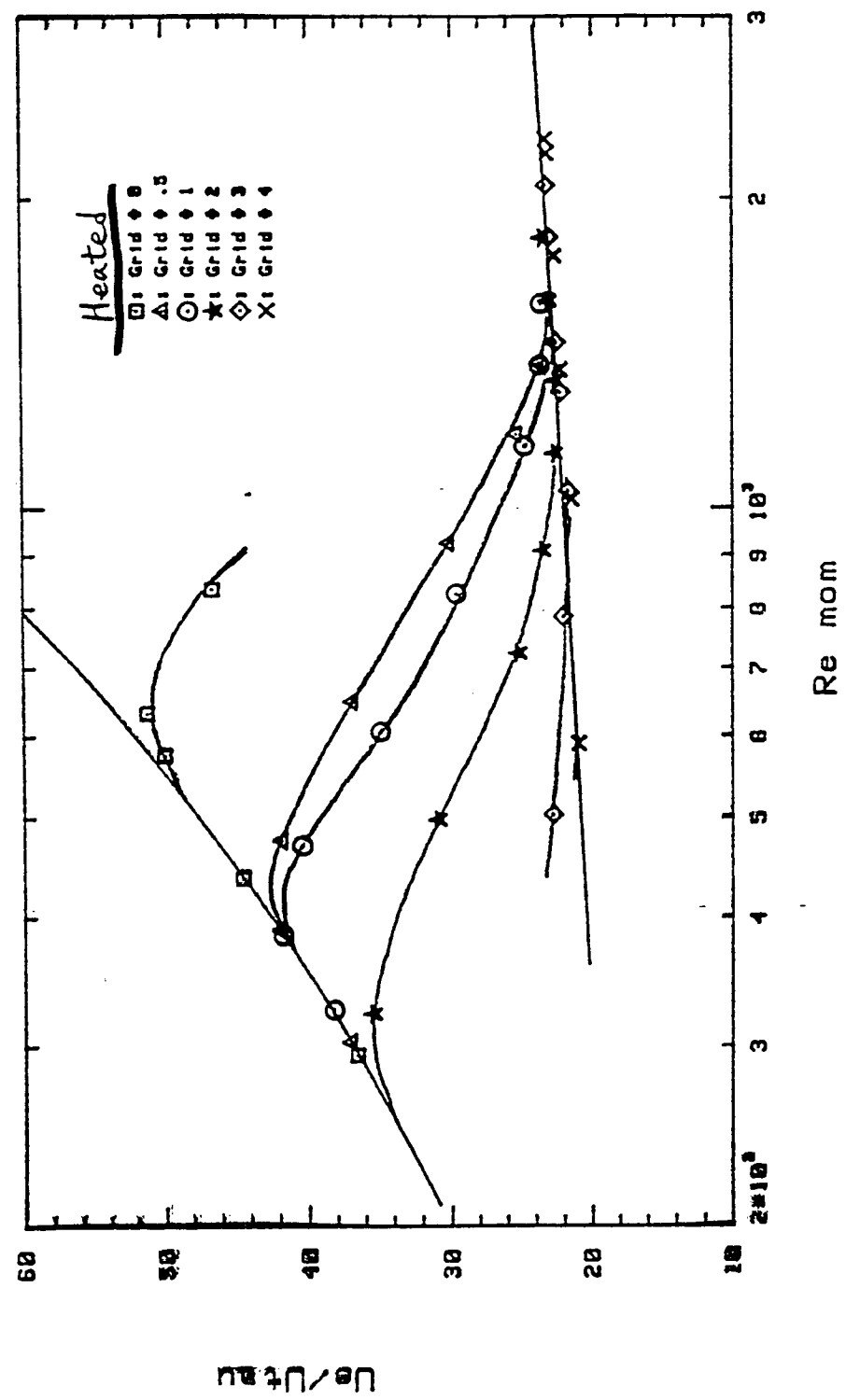


Uet variation



SOHN

$$\frac{u_e}{u_z} = \sqrt{\frac{2}{C_f}}$$



Uet variation

$(\Delta T \sim 15^\circ\text{F})$

Heating <sub>n</sub> reduces  $Re_{x_{tr}}$  by about

20% for Grid 0 (T-s mode)

BUT HAS NEGLIGIBLE EFFECT ON

$Re_{tr}$  FOR THE OTHER GRIDS

(Bypass mode dominant)

---

Note: For distributed roughness

sufficiently large, stability modifiers

are inoperative or may even have

reverse effect. THIS IS ALSO A

BYPASS.

FIG. 1.

SCHEMATIC OF COMPUTATIONAL REGION

(NOT TO SCALE)

GRID 2  
of both Sunder  
and Blair

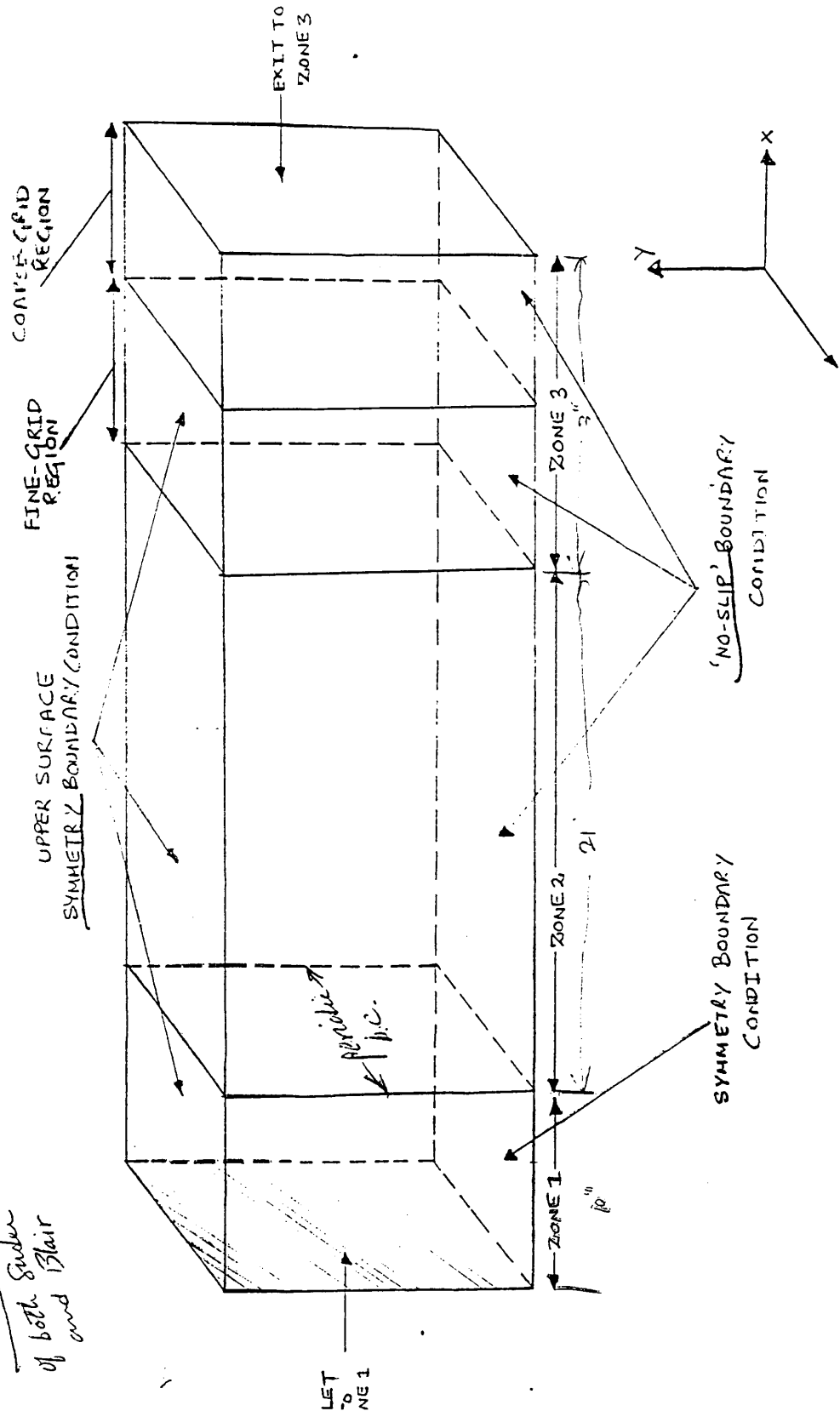
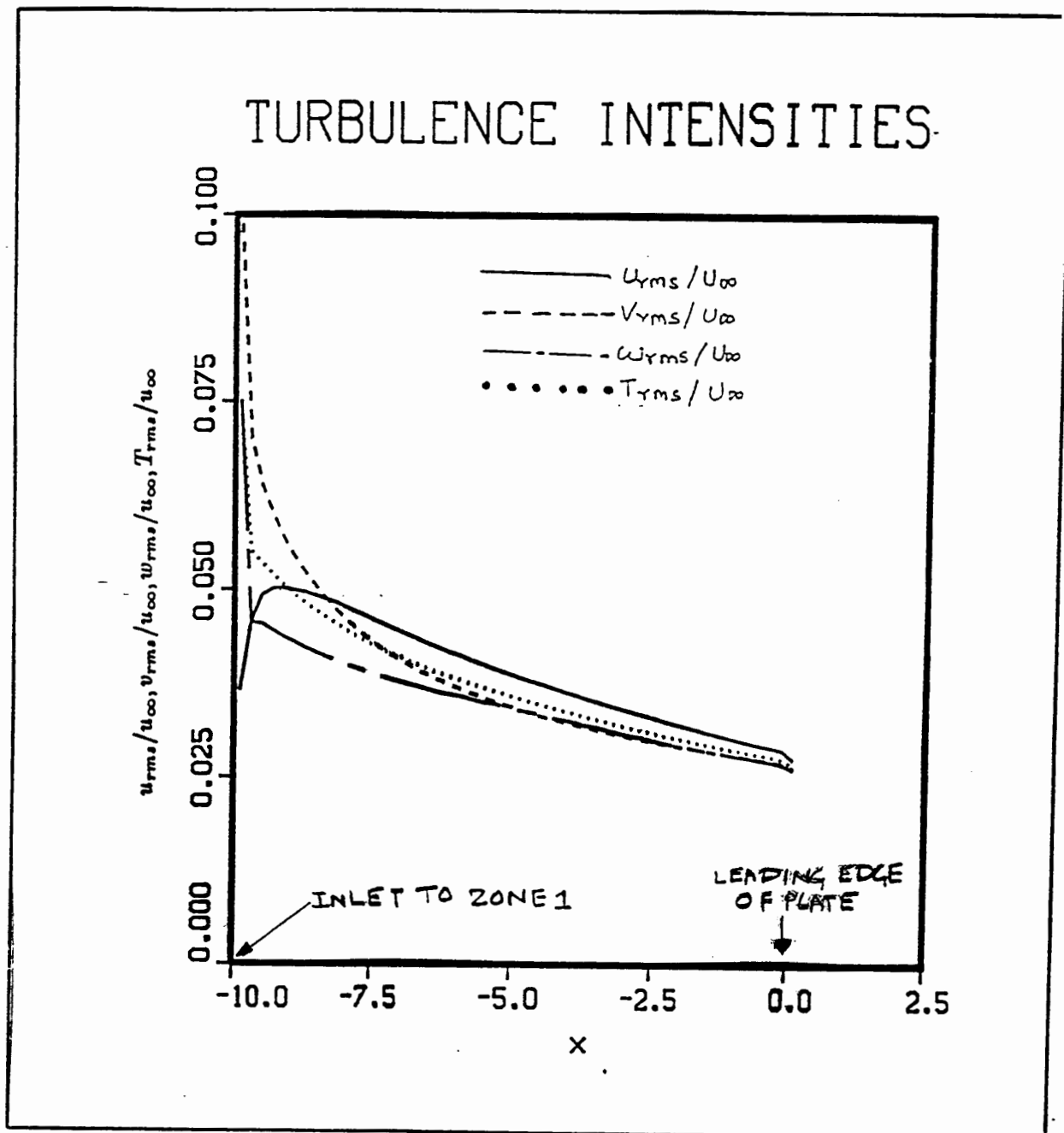
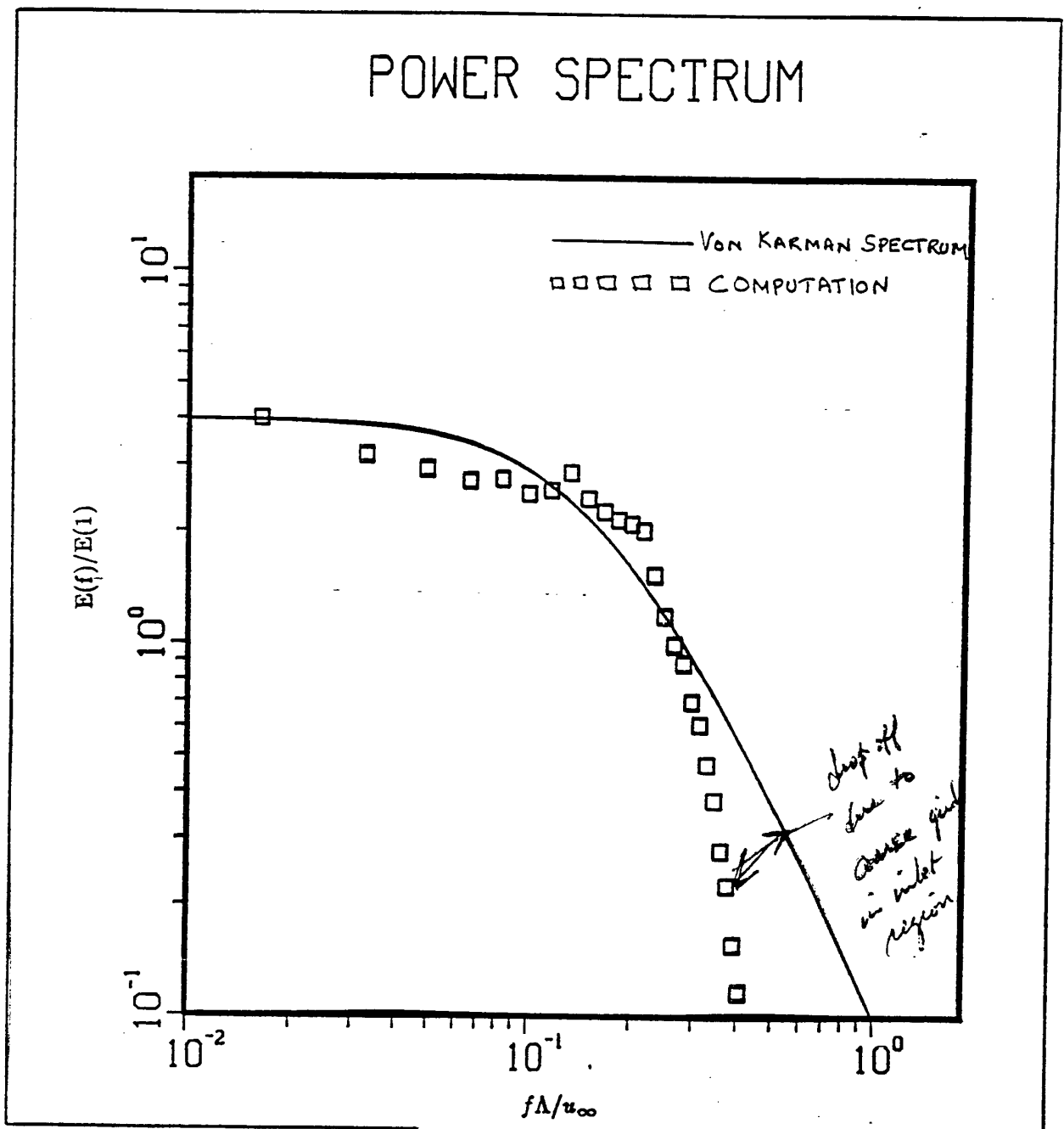


FIG. 2



Rai, 1990

FIG. 3



Rai, 1990

→ Would go a lot faster for  $M=0.4$  rather than  $M=0.1$

FIG 5

→  $16.8 \times 10^6$  grid points  
600-800 hrs  
on CRAY-YMP

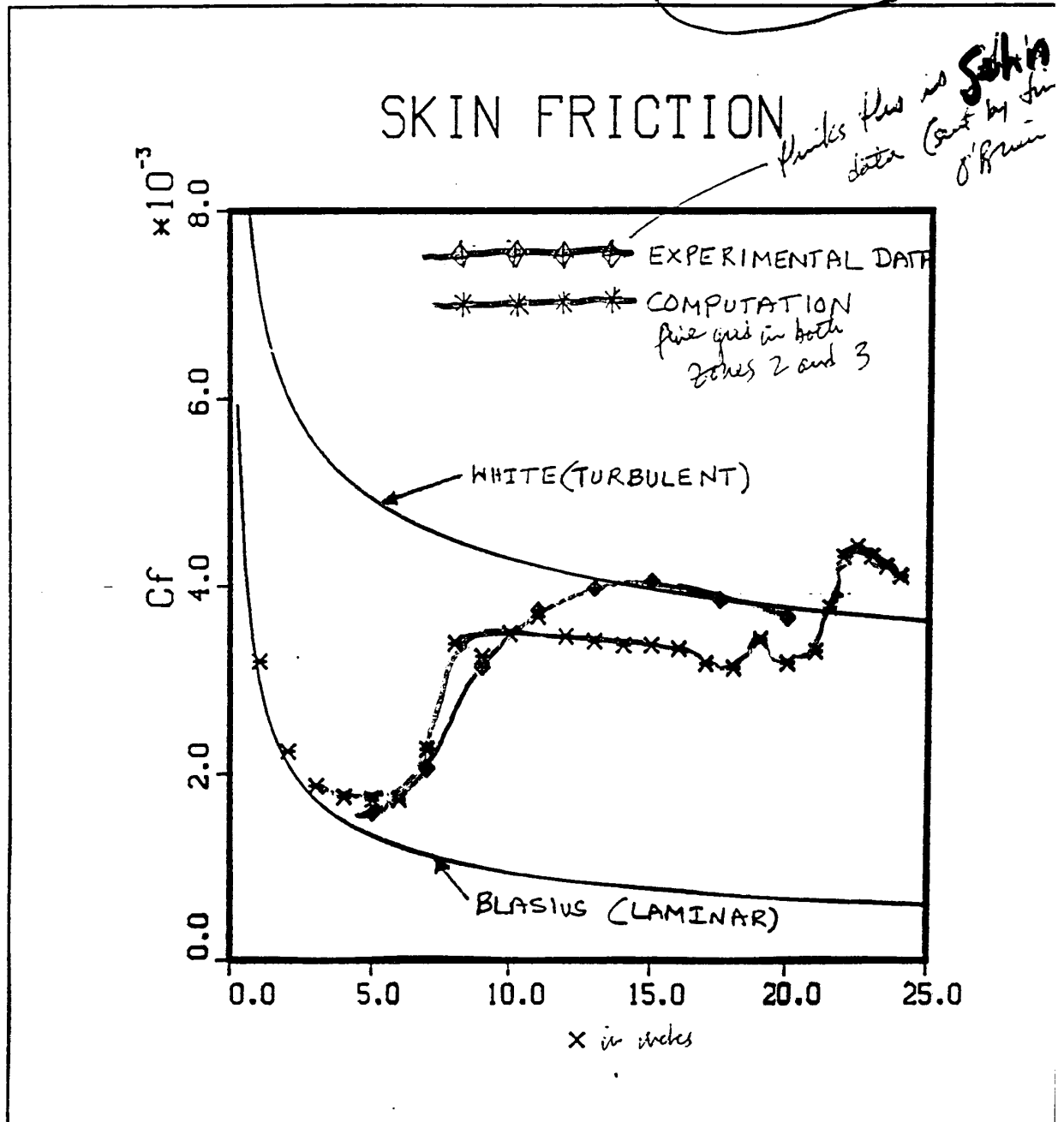


FIG. 6

$\Delta x^+ = 29$   
 $\Delta z^+ = 10$

# Mean Velocity Profile

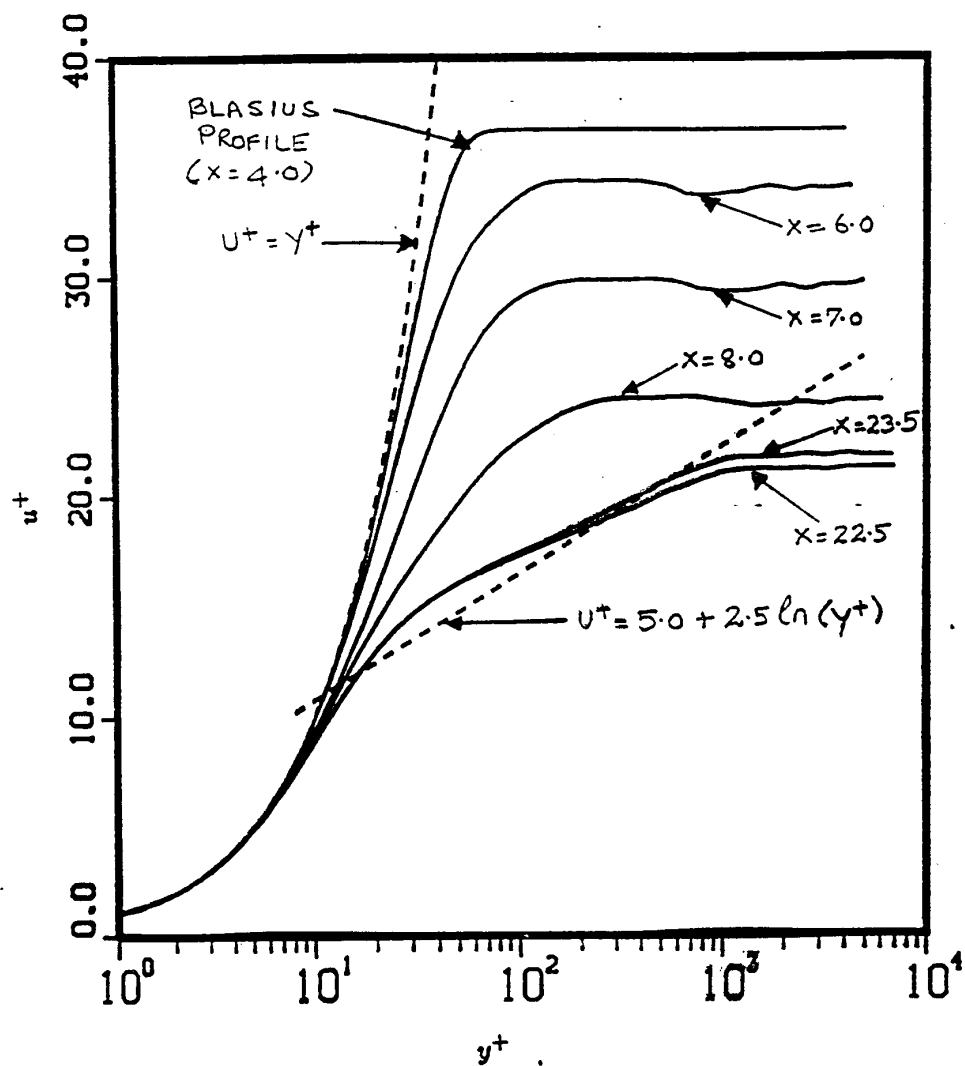
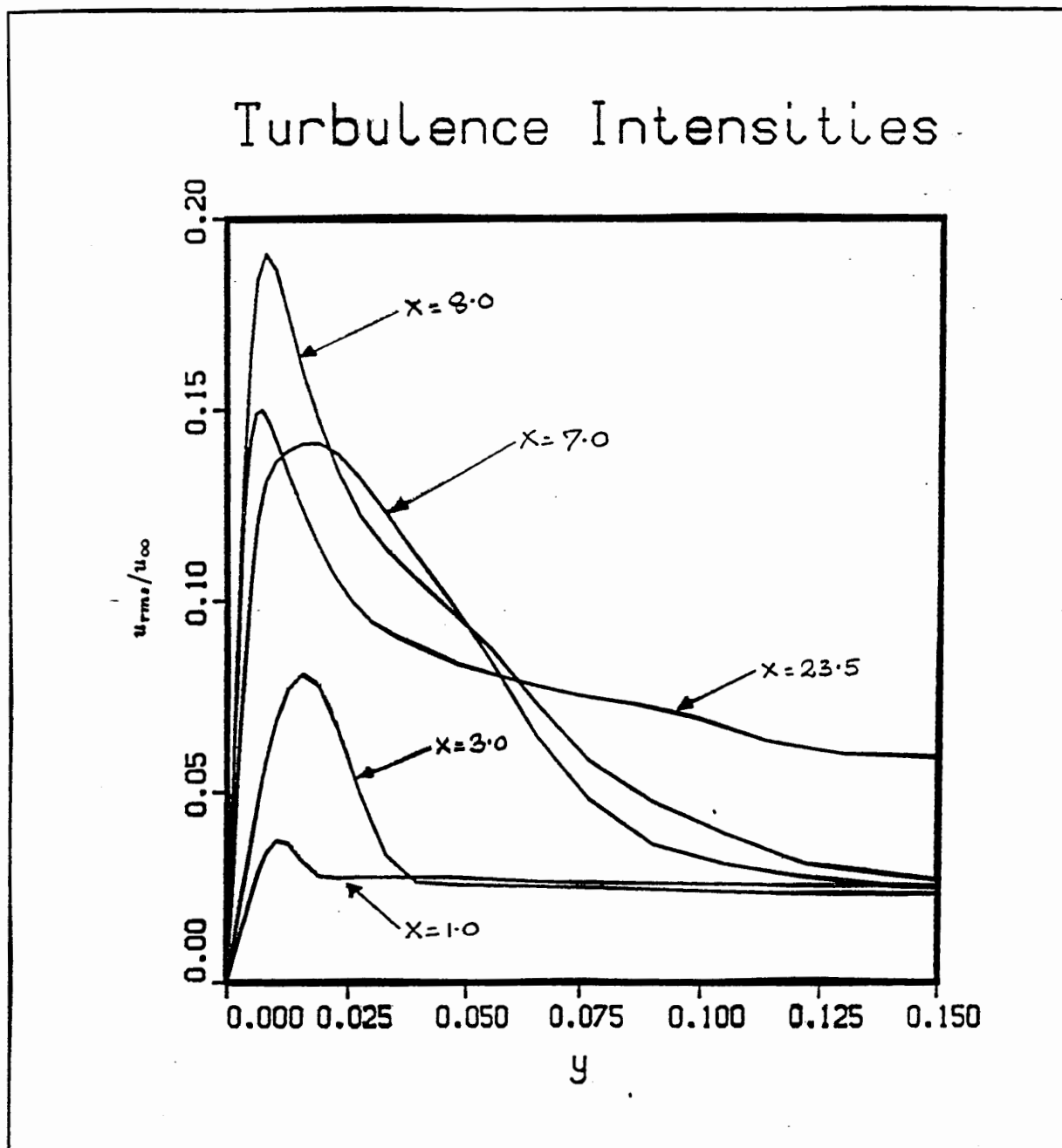


FIG. 7





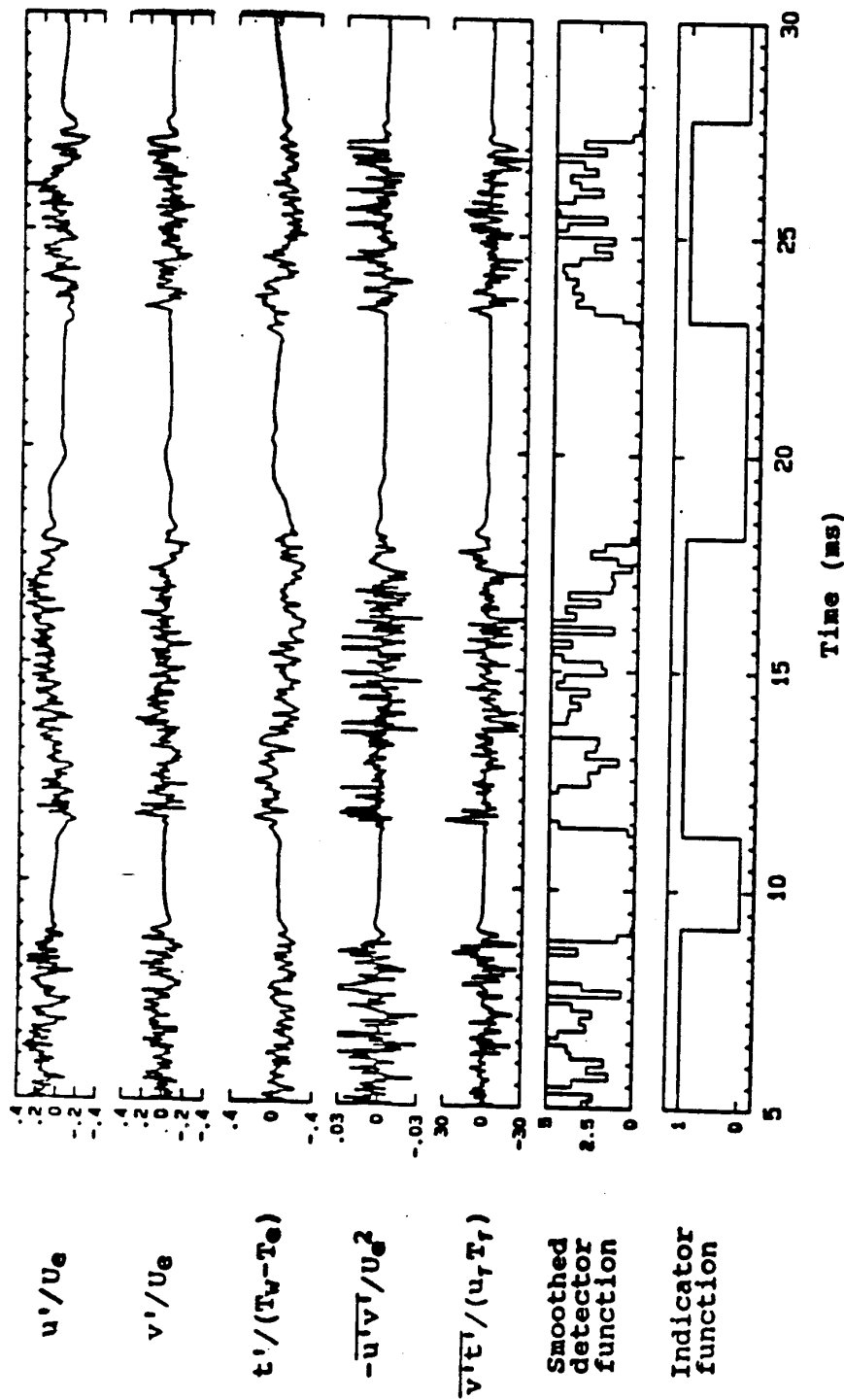


Fig. 18(b) Illustration of indicator function determination technique  
for 3-wire probe following Hedley and Ketter (1974)

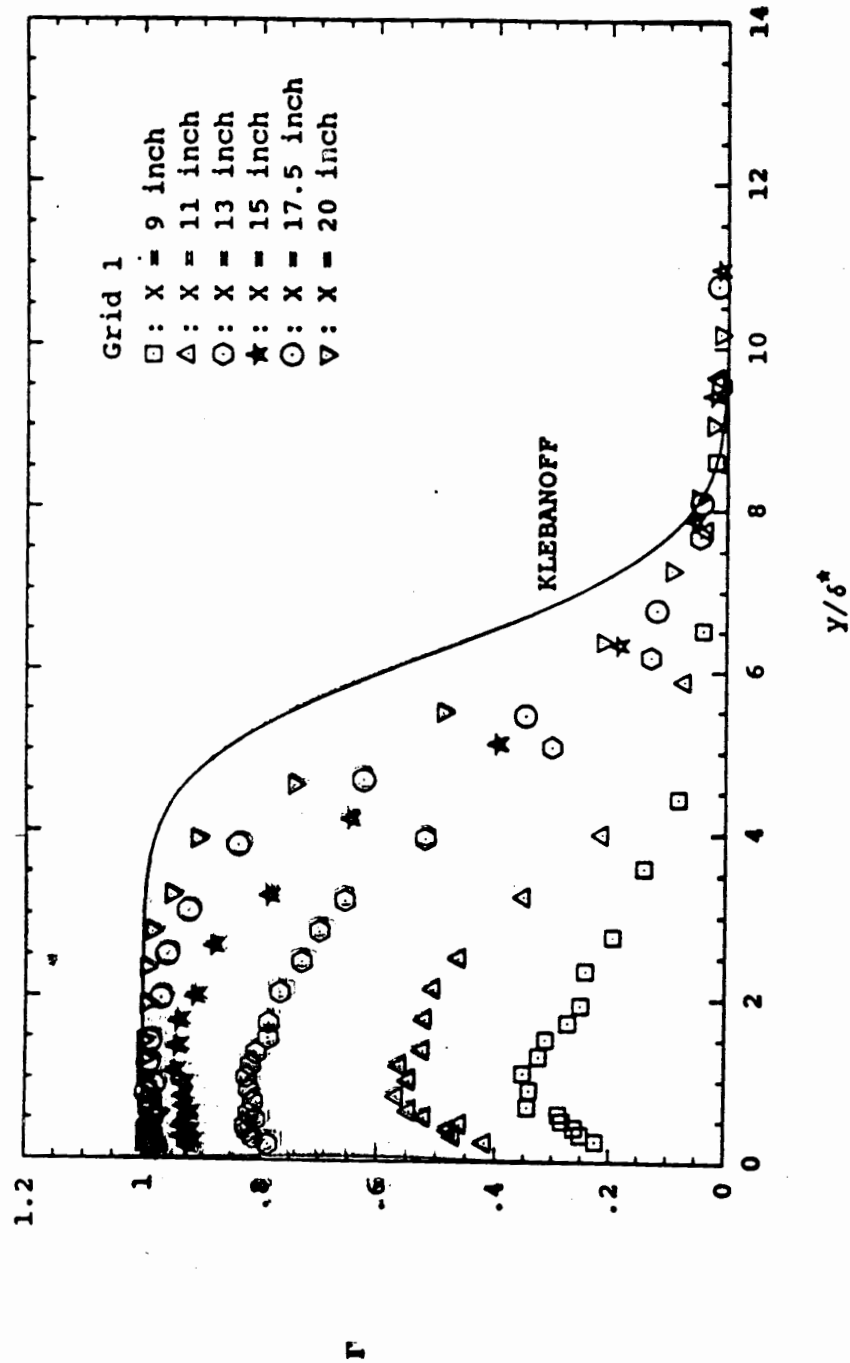


Fig. 28(a) Intermittency profiles across the boundary layer for grid 1

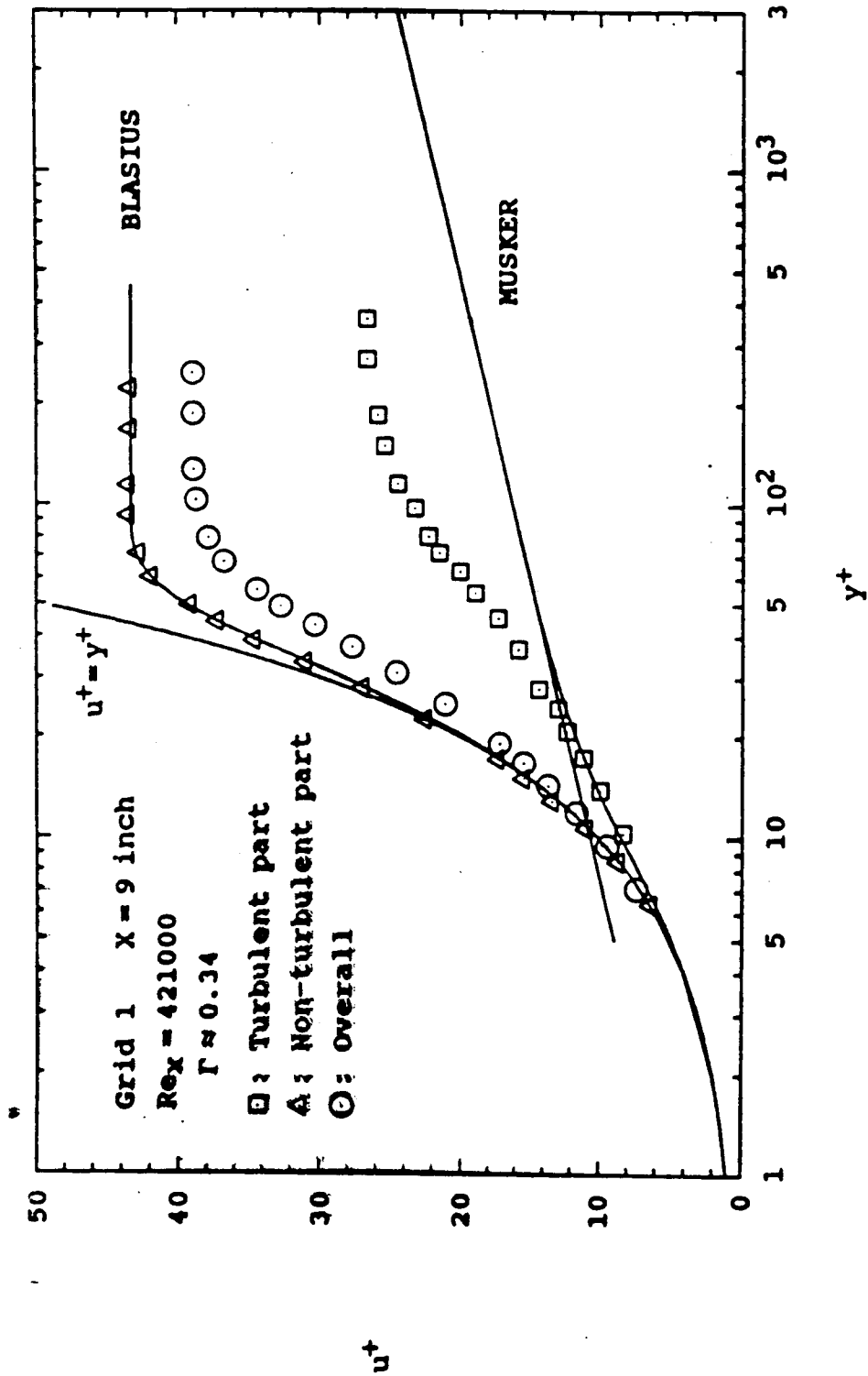


Fig. 29(a) Conditionally sampled mean velocity profiles in wall units at X=9 inches for grid 1

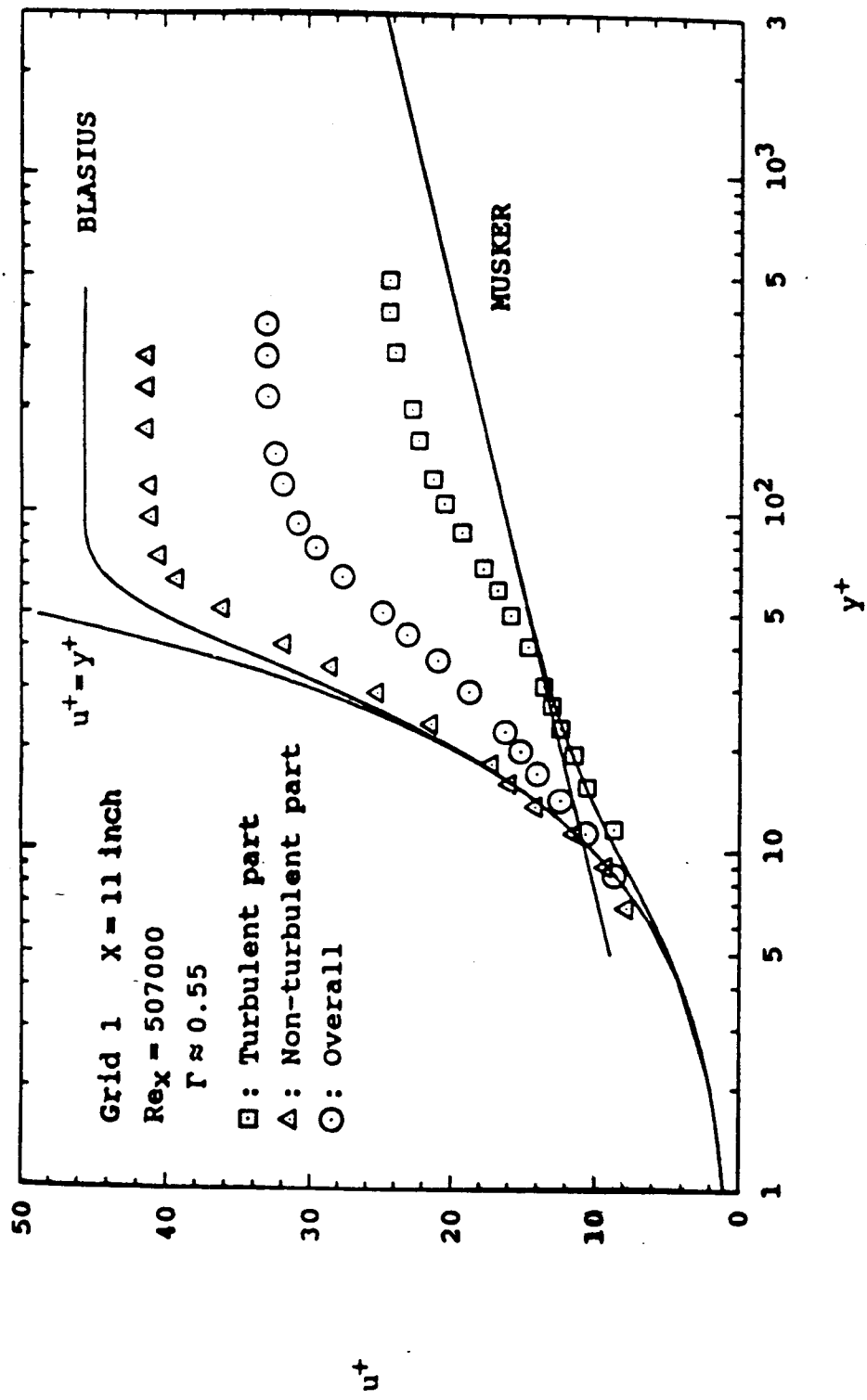
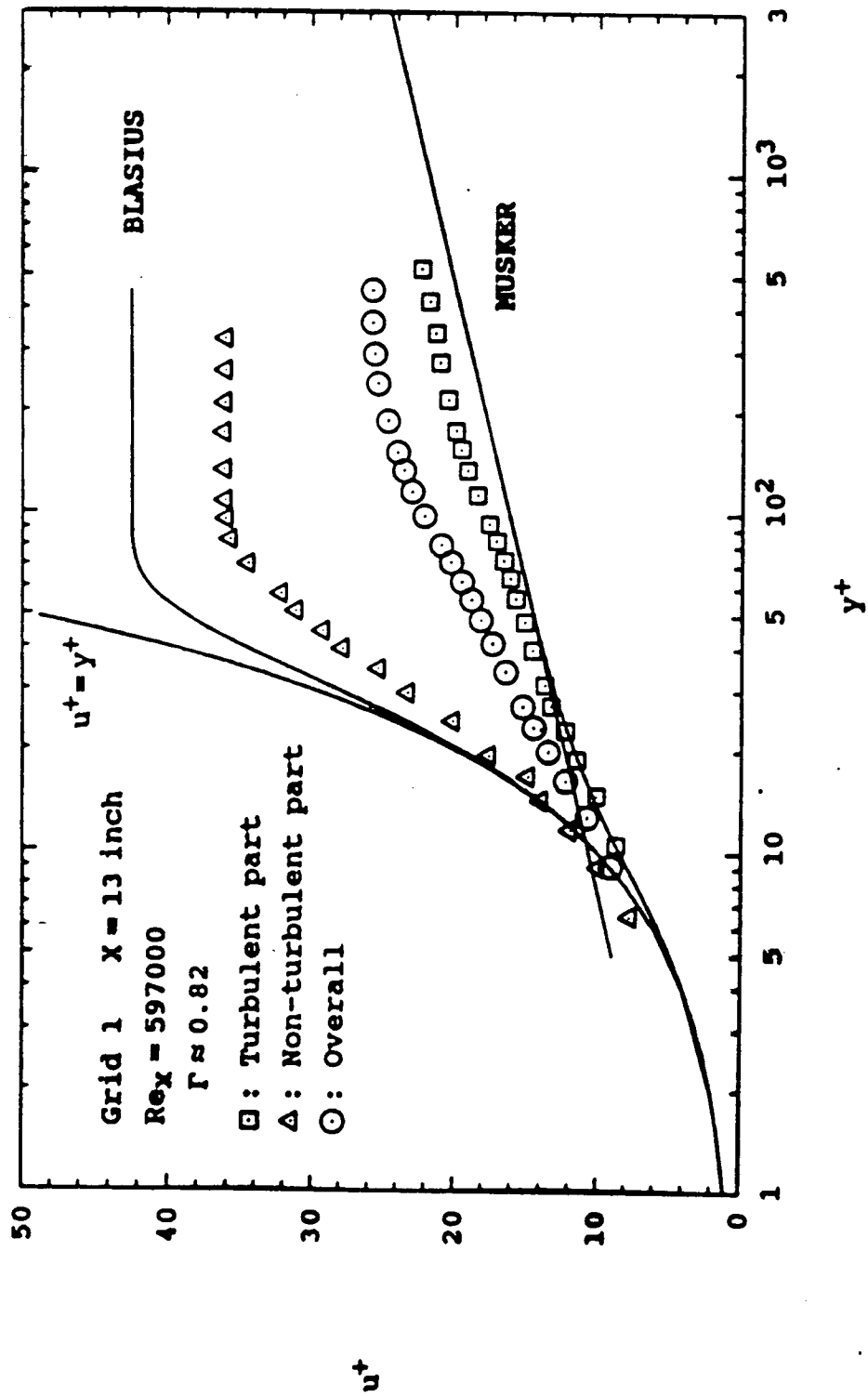


Fig. 29(b) Conditionally sampled mean velocity profiles in wall units at  $X=11$  inches for grid 1



**Fig. 29(c)** Conditionally sampled mean velocity profiles in wall units at  $X=13$  inches for grid 1

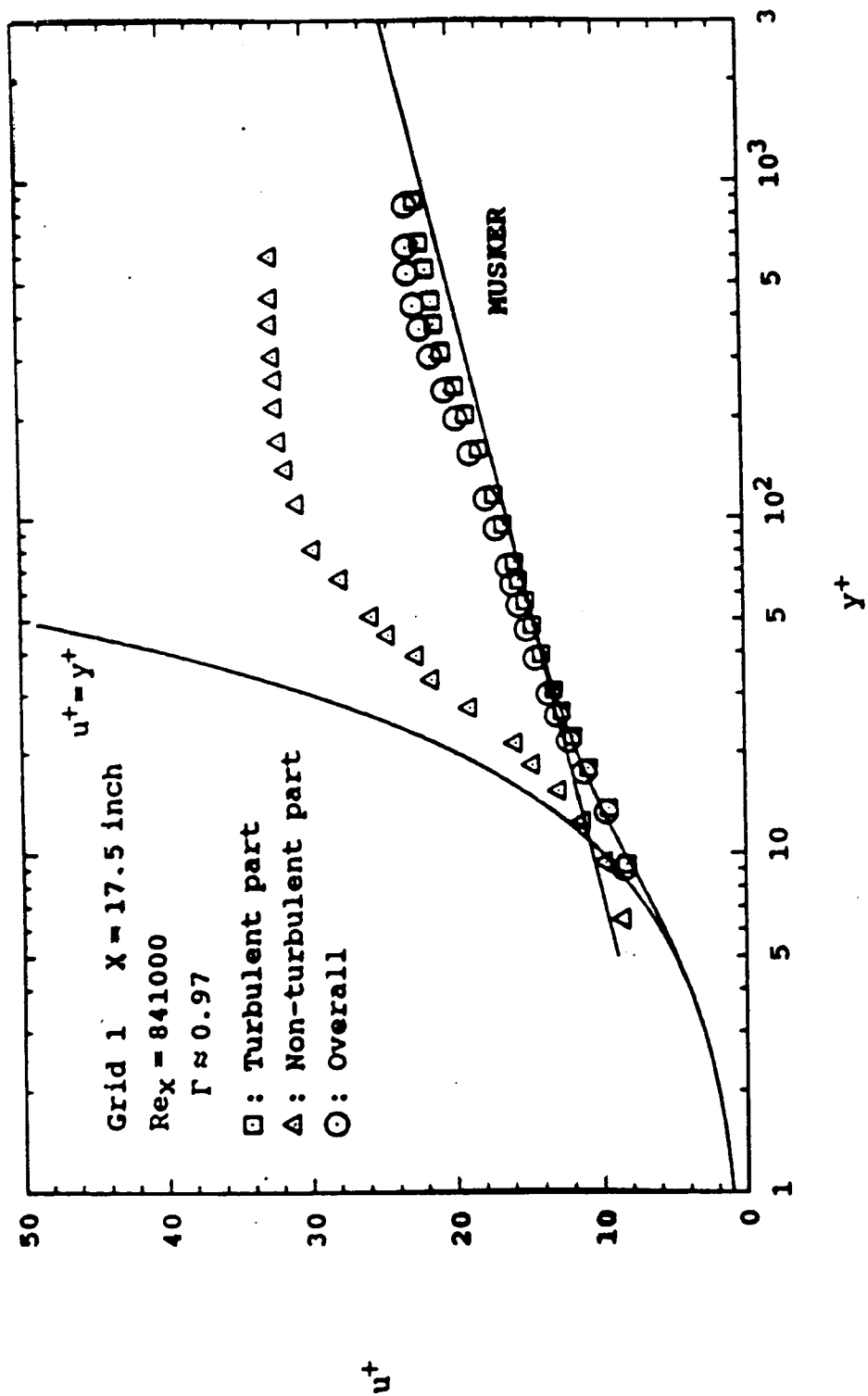


Fig. 29(e) Conditionally sampled mean velocity profiles in wall units at  $X=17.5$  inches for grid 1

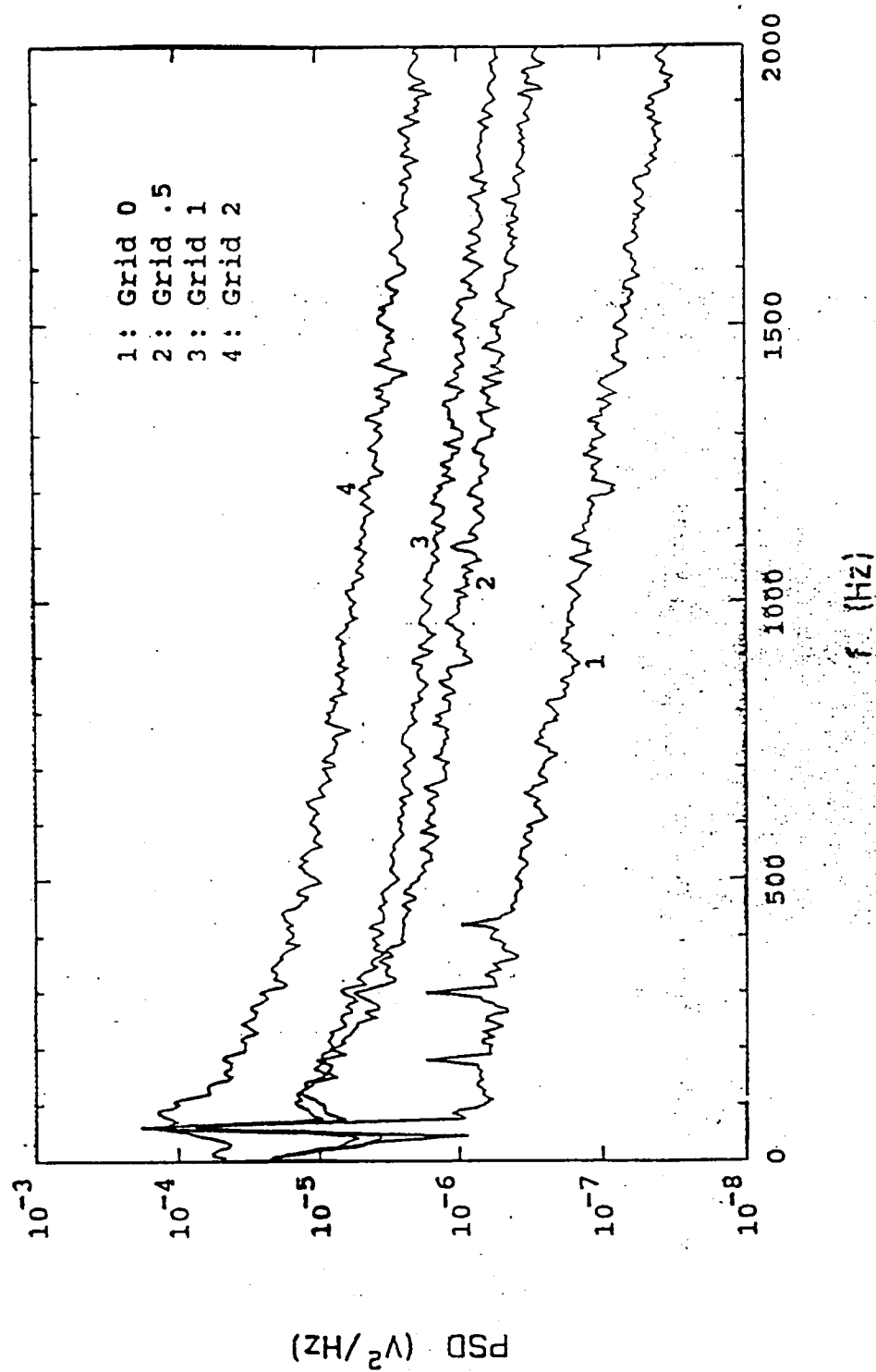


Fig. 54 Prestream spectra for 4 grids

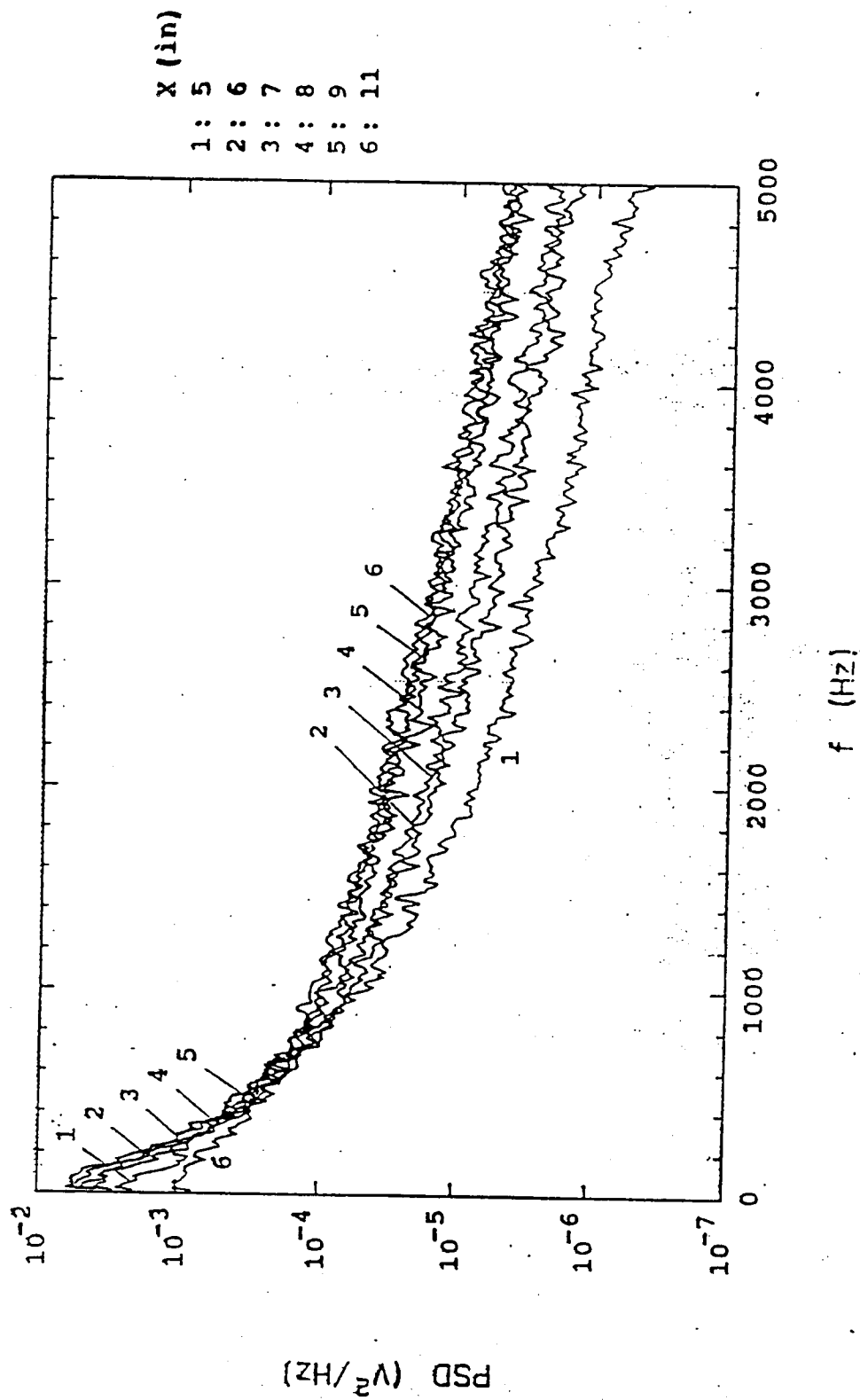


Fig. 53(d) Boundary layer spectra measured at locations where rms velocity reached maximum for grid 2



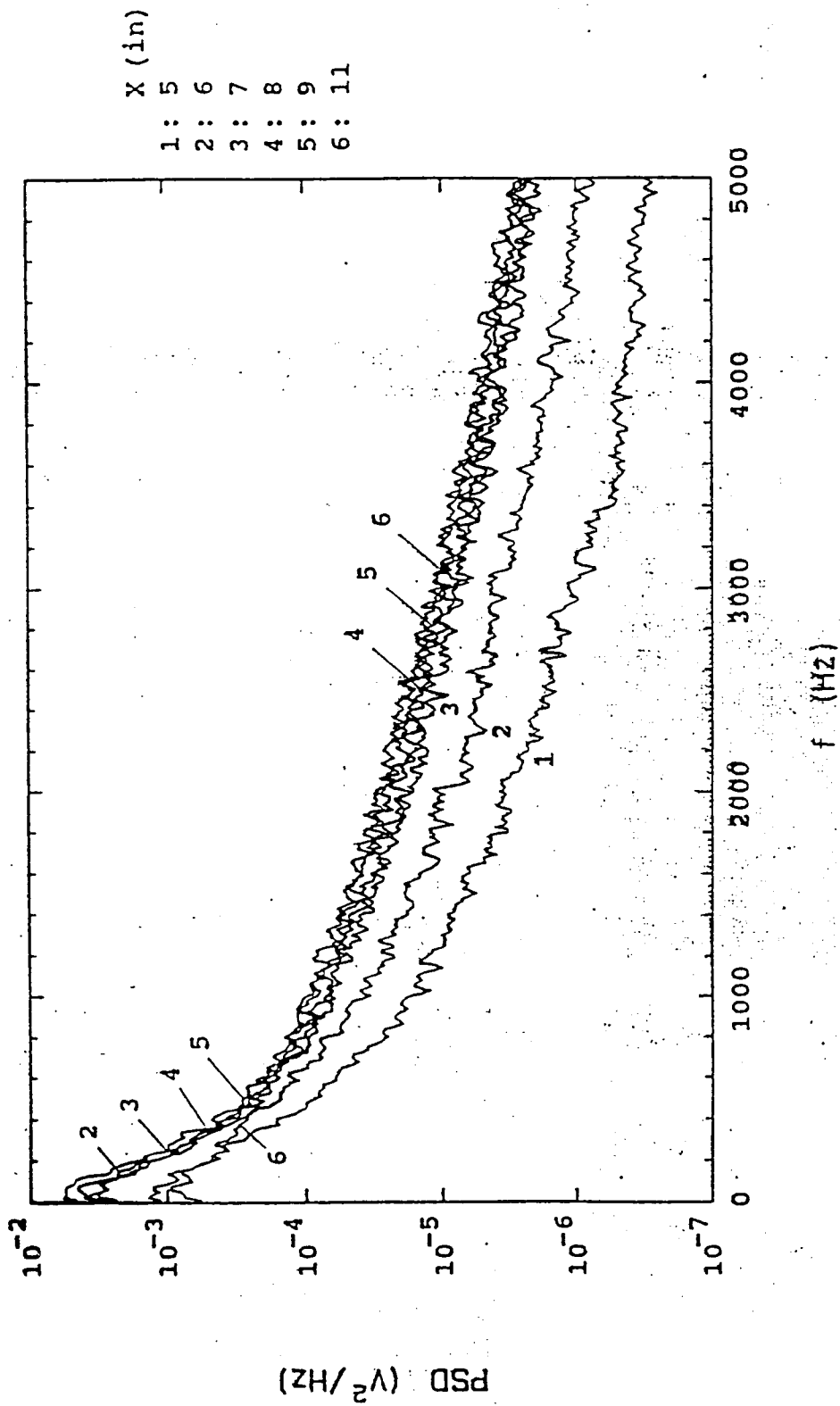


Fig. 52(d) Boundary layer spectra obtained at near-wall locations ( $y_0$ ) for grid 2

THIS SUGGESTS THAT BYPASS  
TRANSITION IN THE CONTEXT OF  
( $Tu$  large)  
GAS TURBINE TECHNOLOGY, MIGHT  
BE MODELED USING TURBULENT  
FLOW METHODS

THESE MUST  
ALL BE VALID  
IN THE  
LAMINAR  
LIMIT

- one-equation models ( $k$ )
  - two-equation models ( $k-\epsilon$ )
  - Reynolds stress models
  - DNS / LES
- 

BUT - HOW DO YOU START THE  
CALCULATION ?

ISSUE : RECEPTIVITY

## K- $\epsilon$ APPROACHES

- Equations should have proper low-Reynolds-number form.
- Turbulence models should have correct near wall behavior for  $k$ ,  $-\overline{u'v'}$ ,  $\epsilon$
- Calculation algorithm should be compatible with equations and turbulence model

STARTING PROFILES FOR

$k$ ,  $\epsilon$  ?

## FEATURES OF BYPASS TRANSITION TO BE MODELED

- Turbulent-like spectra throughout
- Intermittency  $\gamma = \gamma(x, y)$
- With conditional sampling
  - Turbulent part approaches fully-turbulent characteristic as transition progresses
  - Non-turbulent part departs from laminar after transition onset

Intermittency: Non-turbulent part      Turbulent part  
MTS:      Low frequency      High frequency

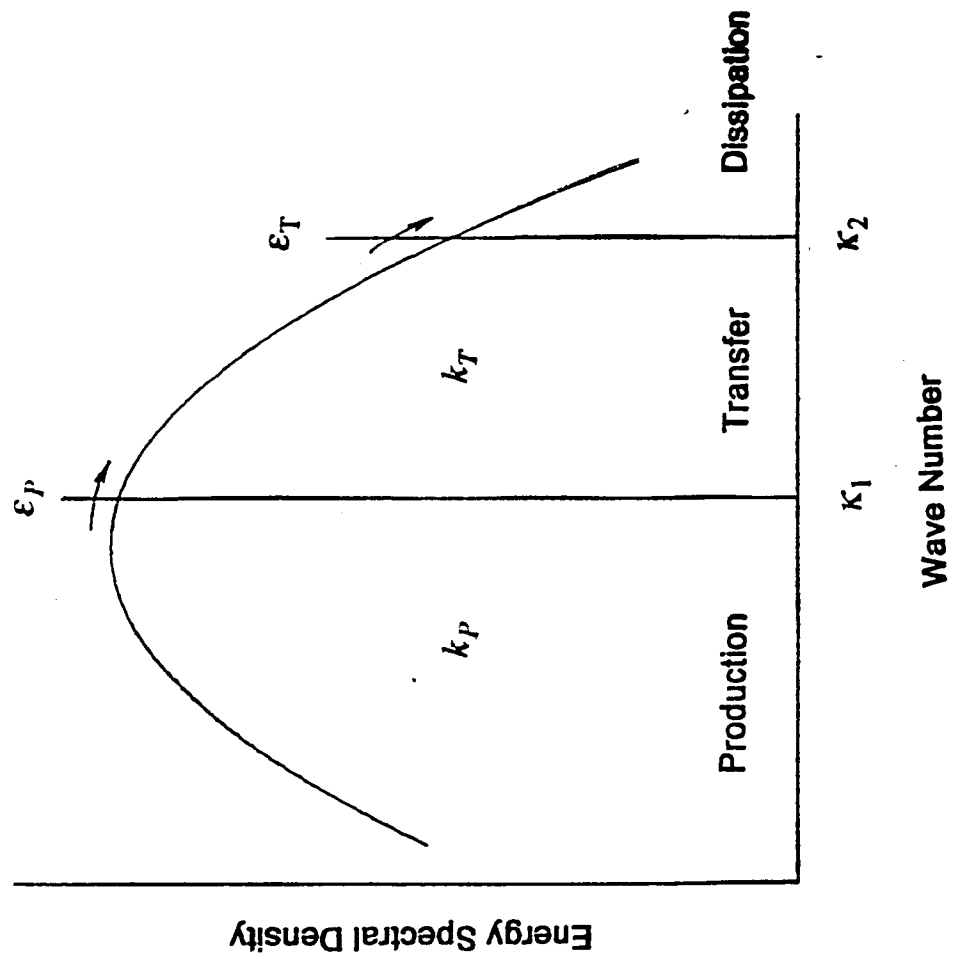
MODEL EACH PART SEPARATELY

DEVELOP INTERMITTENCY PDE (following Kollmann)

COMBINE

USE GIVE UP ON M.T.S.

# Multi-Time-Scale Low Reynolds Number k-ε Turbulence Model



Hanjalic, Launder  
 & Schiestel, 1980

# Multi-Time-Scale Low Reynolds Number k-ε Turbulence Model

S-Y WU

$$\begin{aligned}
 U \frac{\partial k_P}{\partial x} + V \frac{\partial k_P}{\partial y} &= \frac{\partial}{\partial y} \left[ \left( \nu + \frac{\nu_t}{\sigma_k} \right) \frac{\partial k_P}{\partial y} \right] + P_k - \varepsilon_P \\
 U \frac{\partial k_T}{\partial x} + V \frac{\partial k_T}{\partial y} &= \frac{\partial}{\partial y} \left[ \left( \nu + \frac{\nu_t}{\sigma_k} \right) \frac{\partial k_T}{\partial y} \right] + \varepsilon_P - \varepsilon_T \\
 U \frac{\partial \varepsilon_P}{\partial x} + V \frac{\partial \varepsilon_P}{\partial y} &= \frac{\partial}{\partial y} \left[ \left( \nu + \frac{\nu_t}{\sigma_\varepsilon} \right) \frac{\partial \varepsilon_P}{\partial y} \right] + C_{P1} \left[ \left( 1 + C_{P1} \frac{P_k}{\varepsilon_P} \right) \right] P_k \frac{\varepsilon_P}{k_P} - C_{P2} f_{P2} \frac{\varepsilon_P^2}{k_P} \\
 U \frac{\partial \varepsilon_T}{\partial x} + V \frac{\partial \varepsilon_T}{\partial y} &= \frac{\partial}{\partial y} \left[ \left( \nu + \frac{\nu_t}{\sigma_\varepsilon} \right) \frac{\partial \varepsilon_T}{\partial y} \right] + C_{T1} \left[ \left( 1 + C_{T1} \frac{\varepsilon_P}{\varepsilon_T} \right) \right] \varepsilon_P \frac{\varepsilon_T}{k_T} - C_{T2} f_{T2} \frac{\varepsilon_T^2}{k_T}
 \end{aligned}$$

**where**       $P_k = \nu_t \left( \frac{\partial U}{\partial y} \right)^2$        $\nu_t = c_\mu f_\mu \frac{k^2}{\varepsilon_P}$        $k = k_P + k_T$

**at wall**       $k_P = k_T = 0$        $\varepsilon_P = 2\nu \left( \frac{\partial \sqrt{k_P}}{\partial y} \right)^2$        $\varepsilon_T = 2\nu \left( \frac{\partial \sqrt{k}}{\partial y} \right)^2$

## Multi-Time-Scale Low Reynolds Number k-ε Turbulence Model

- Patankar-Spalding method:

Grid point  $N=102$

Step size  $\Delta x=0.1\delta_2$

Non-uniform grid  $\Delta\omega_\eta=1.02\Delta\omega_{\eta-1}$

- Inlet condition:

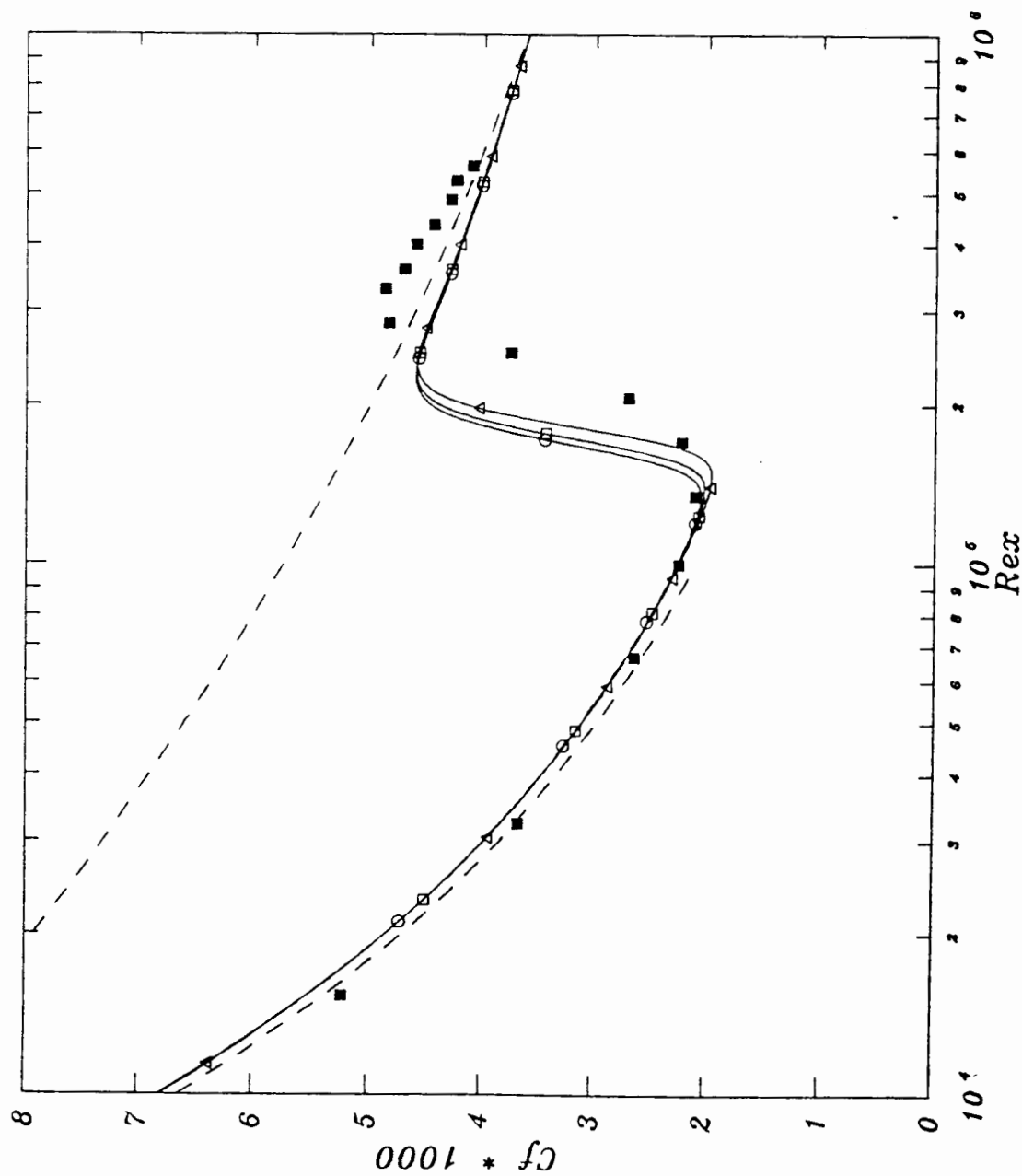
$Re_x=100$

$U=Blasius$  solution

$k=Reshotko's$  profile  $k = \frac{1}{2}U^2Tu^2\left(f' + \frac{\eta}{2}f''\right)^2$

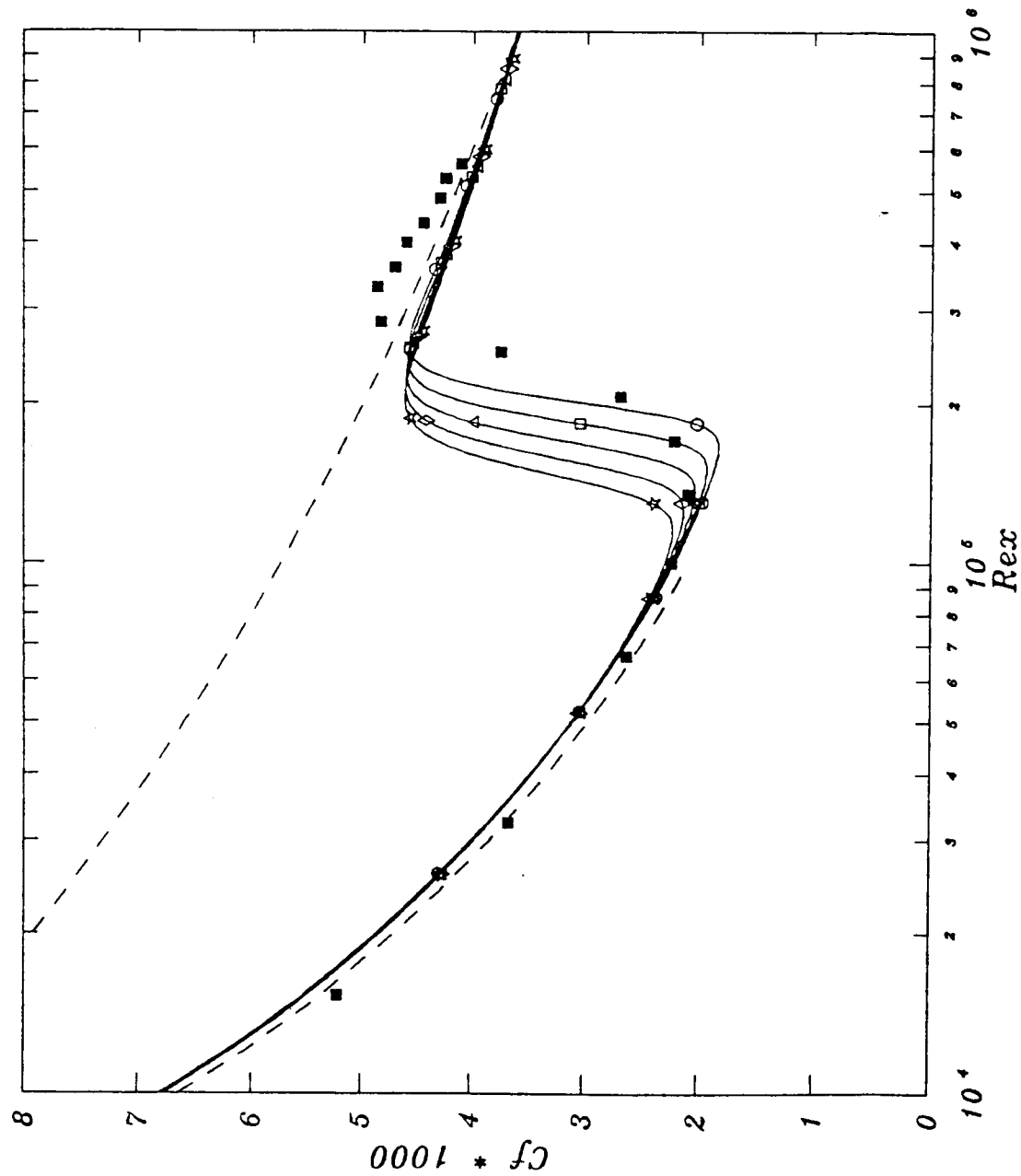
$\epsilon=Rodi's$  profile  $\epsilon_P = a_1k_P \frac{\partial U}{\partial y}$   $\epsilon_T = a_1k_T \frac{\partial U}{\partial y}$

INFLUENCE OF THE STARTING LOCATION  
Initial profiles:  $K$  and  $\varepsilon$  are Rodi's profiles  
Freestream:  $K_p=0.7K_e$

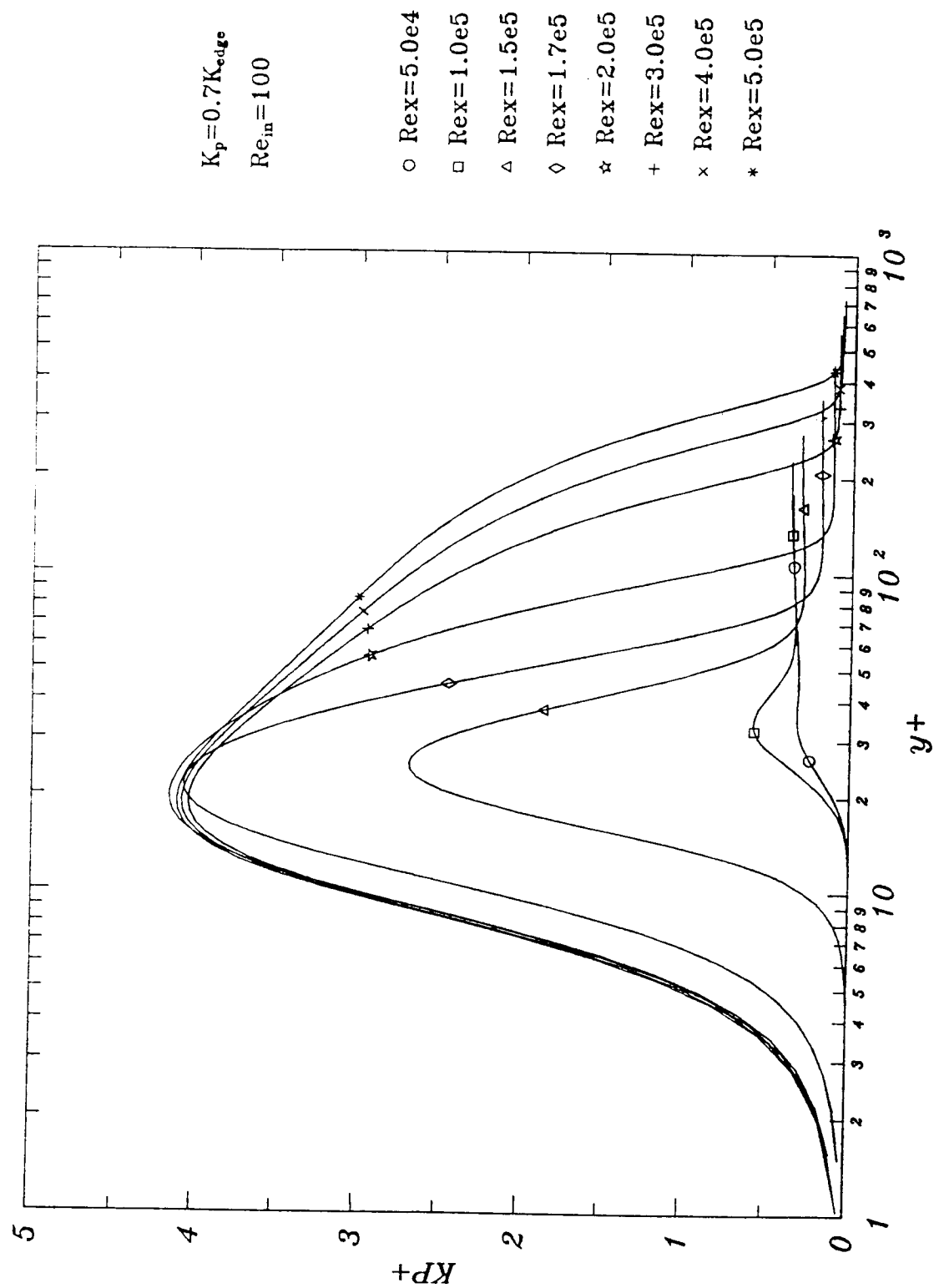




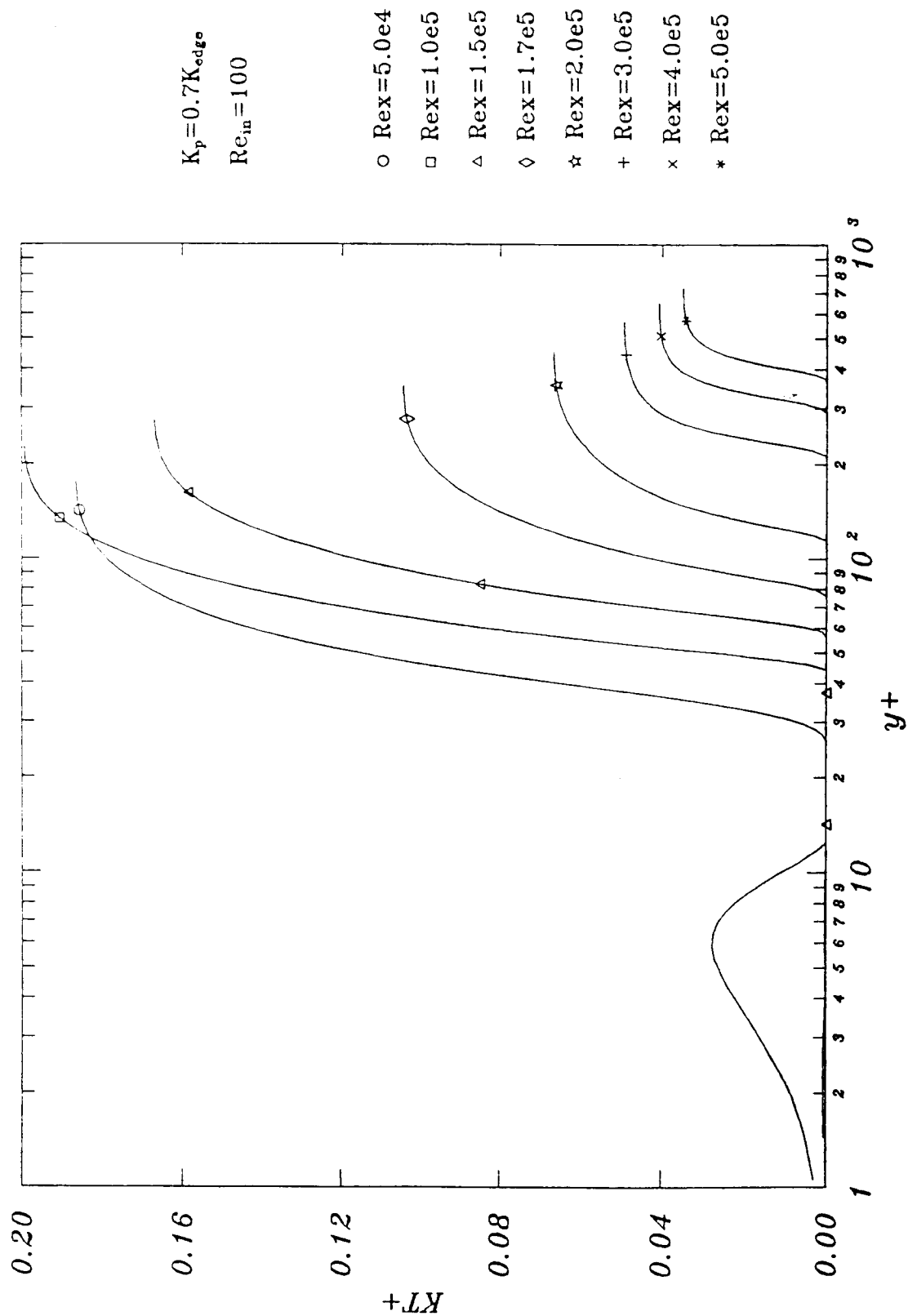
# INFLUENCE OF THE PARTITION OF $K_p$ AND $K_t$ Initial condition: $K$ =Reshotko's profile, $\varepsilon$ =Rodi's profile



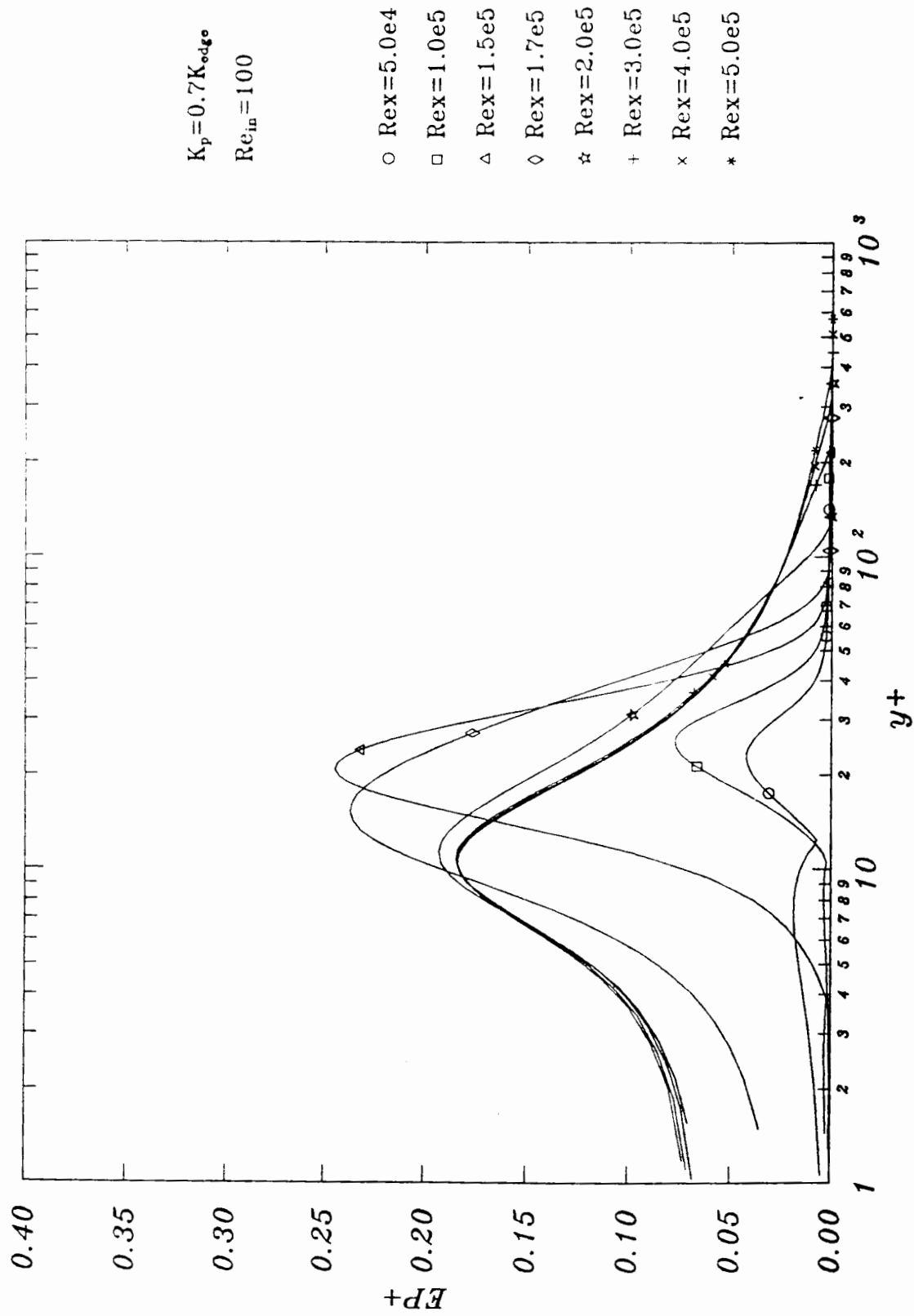
PRODUCTION ZONE TURBULENT KINETIC ENERGY,  $K_p^+$ ,  
 Multi-time-scale  $k-\varepsilon$  Model  
 ERCOFTAC Test Case T3A



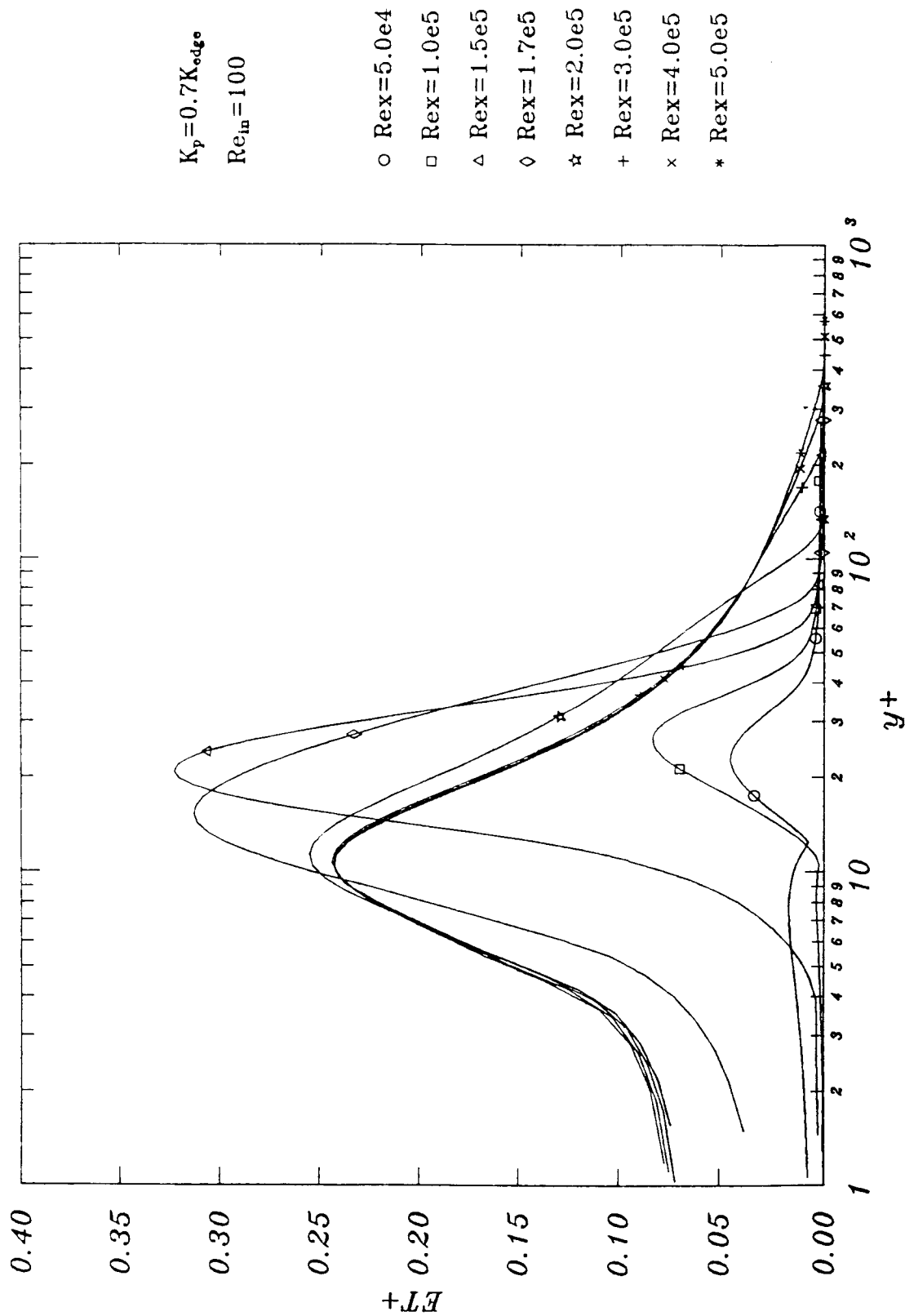
TRANSFER ZONE TURBULENT KINETIC ENERGY,  $K_T^+$ ,  
 Multi-time-scale  $k-\varepsilon$  Model  
 ERCOFTAC Test Case T3A



ENERGY TRANSFER FUNCTION,  $E_p^+$ ,  
Multi-time-scale  $k-\varepsilon$  Model  
ERCOFTAC Test Case T3A

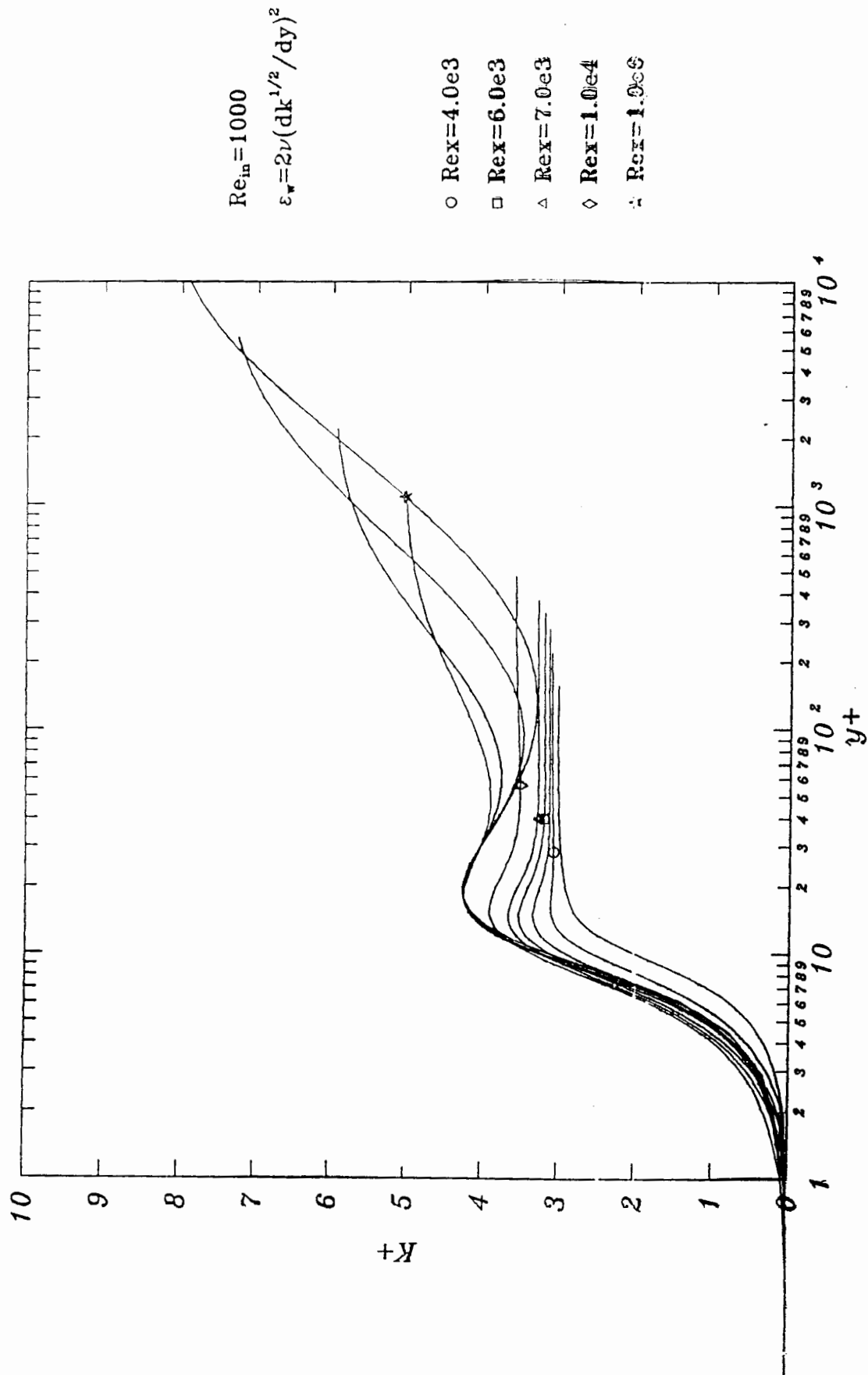


# DISSIPATION, $E_T^+$ , Multi-time-scale $k-\varepsilon$ Model ERCOFTAC Test Case T3A

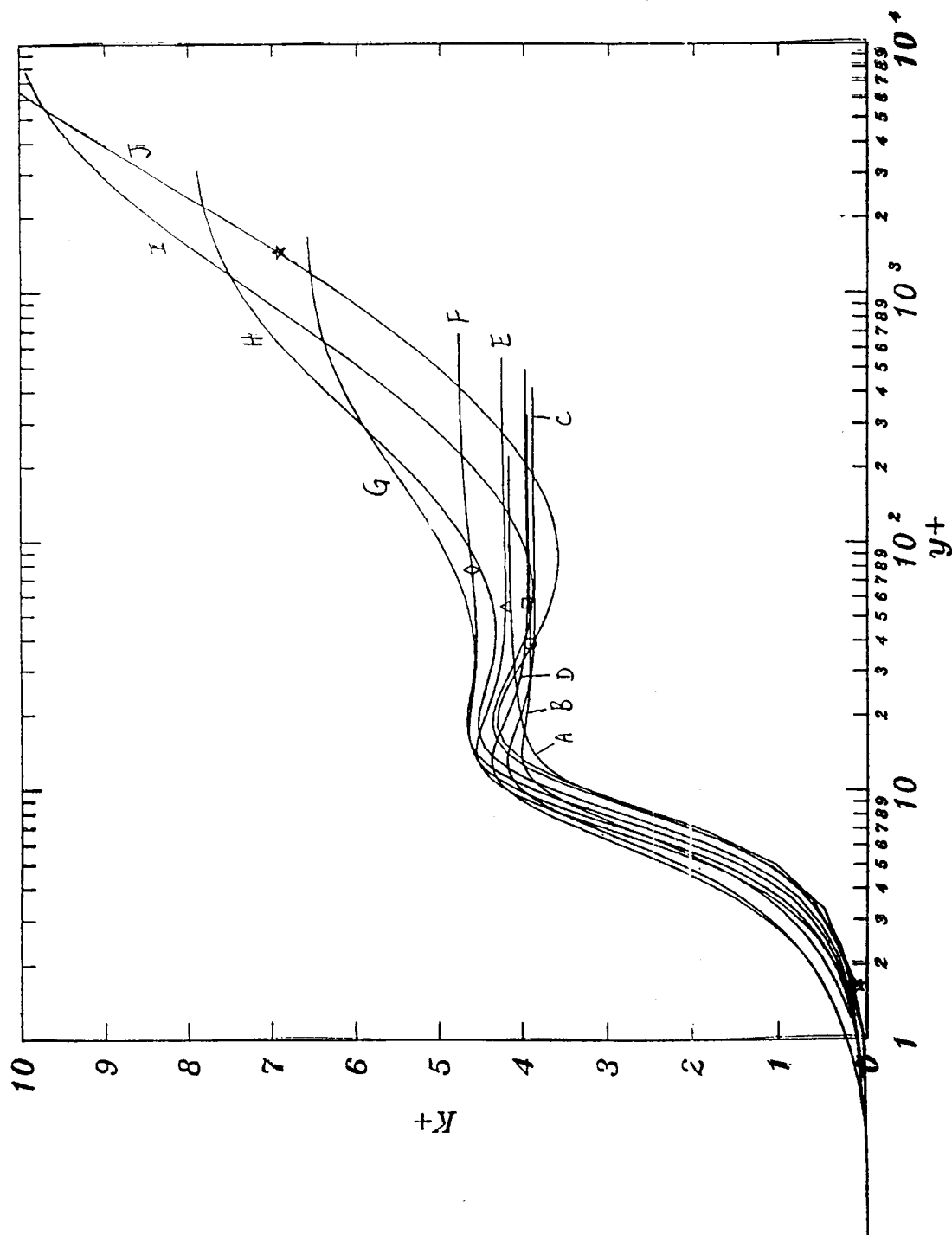


TURBULENT KINETIC ENERGY,  $K^+$ ,  
Multi-time-scale  $k-\varepsilon$  Model  
High Free-Stream Turbulence, 12 percents

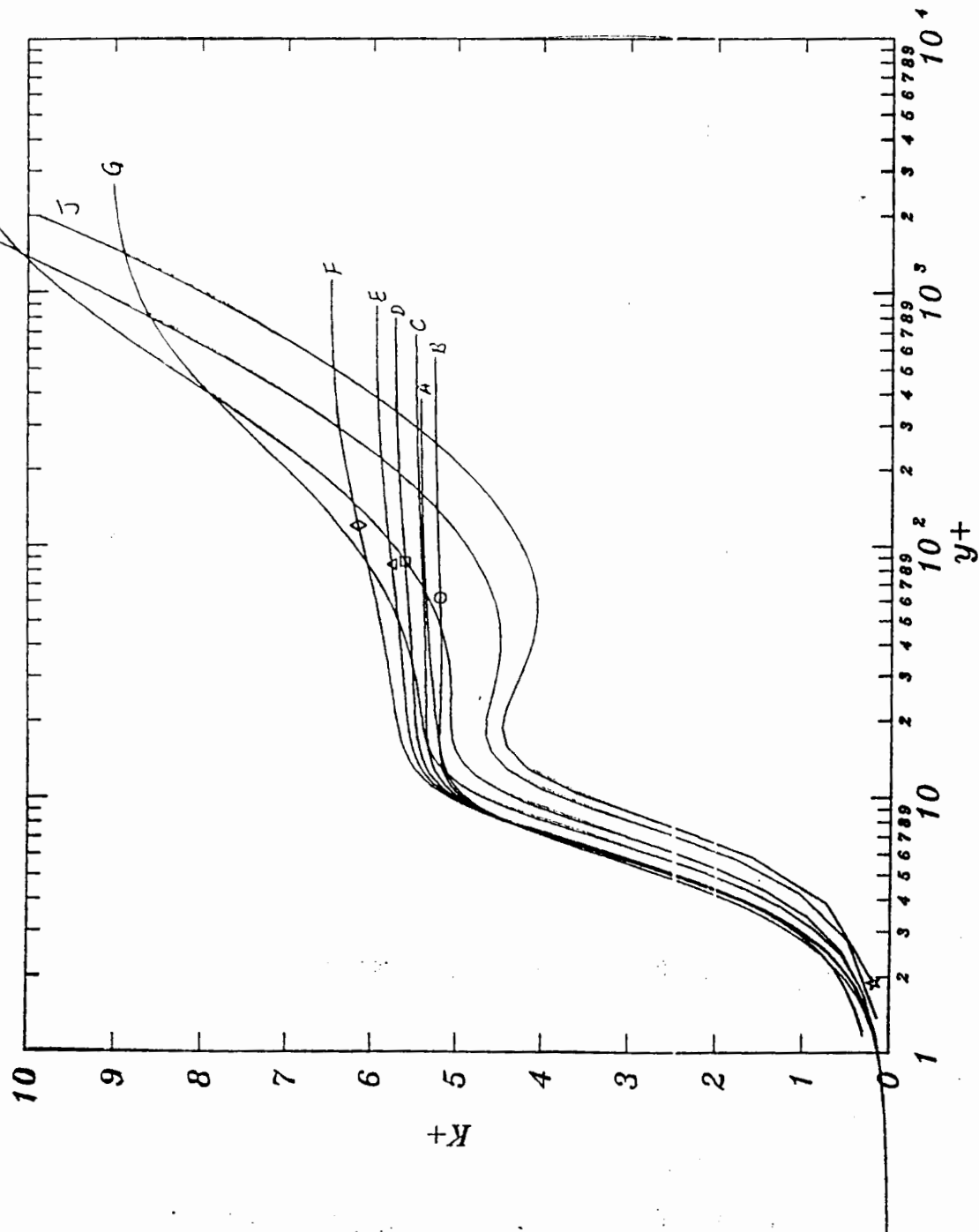
$$\varepsilon_{in} = 798.$$



TURBULENT KINETIC ENERGY,  $K^+$ ,  
Multi-time-scale  $k-\varepsilon$  Model  
High Free-Stream Turbulence, 15 percents



TURBULENT KINETIC ENERGY,  $K^+$ ,  
Multi-time-scale  $k-\varepsilon$  Model  
High Free-Stream Turbulence, 20 percents





# DIRECTIONS

- CONTINUE EXPERIMENTAL STUDIES TO PROVIDE PHENOMENOLOGICAL DATA FOR MODELING VALIDATION
- REFINE MODELS (MTS, etc) TO BE USED FOR TURBINE BLADE DESIGN CALCULATIONS
- LOOK HARD AT "RECEPTIVITY" ISSUE TOWARD IDENTIFYING PHYSICS OF BYPASS INITIATION

EXPERIMENTS — BREUER, KENDALL

ANALYSIS — LANDAHL, HENNINGSON  
COMPUTATION SCHMID  
HERBERT (PSE)

